



Workshop on Computability Theory 2010.
Session I: Ponta Delgada (Azores), Portugal.

PROGRAM

Monday, July 5

10:00-10:45 **Jan Reimann**, *Algorithmic Equivalence Relations*.

Borel equivalence relations arising from recursion theoretic reducibilities seem to exhibit a stubborn resistance to complete classification. One reason may be seen in the fact that they relate to deep recursion theoretic problems, first and foremost Martin's Conjecture on degree invariant functions. I will present some recent progress on the classification of some recursion theoretic equivalence relations.

11:00-11:45 **Damir Dzhafarov**, *Reverse mathematics and equivalents of the axiom of choice*.

We investigate the effective and proof theoretic content of various propositions which are classically equivalent to the axiom of choice. Formalizing these in the language of second order arithmetic gives rise to principles of varying strength: some are provable in RCA_0 , some are equivalent to ACA_0 , and for each standard $n \geq 1$ some are equivalent to $\Pi_n^1-CA_0$. We focus on the principle *FIP*, which asserts that every family of nonempty sets has a maximal subfamily with the property that any finite intersection of its members is nonempty. This principle is incomparable with WKL_0 , and modulo Σ_2^0 induction lies in between the model-theoretic principles *AMT* and *OPT* studied by Hirschfeldt, Slaman, and Shore [2010]. This is joint work with Carl Mummert.

12:00-12:45 **Peter Koepke**, *Infinite Time Computations*.

The computations of the Infinite Time Turing Machines of Hamkins et al are carried out along an unbounded ordinal time axis. At each point in time there is a Turing tape configuration. At successor ordinals, the configuration is obtained from the predecessor configuration as in ordinary Turing computations. At limit times, some appropriate limit of previous configurations is taken. This general idea leads to Ordinal Turing Machines or Ordinal Register Machines, when one employs ordinal sized tapes or registers instead of standard Turing tapes. We shall describe those machines and also consider "Infinite Time Blum Shub Smale Machines" operating on real or complex numbers.

14:45-15:30 **Andrea Sorbi**, *The Medvedev lattice and intermediate logics*.

15:45-16:30 **Elvira Mayordomo**, *Effective dimension and randomness for languages and pairs of languages*.

In this talk I will survey different levels of both effective dimension and randomness ranging from finite-state to constructive. I will also look at the extension from languages to pairs of languages ending with the application to NP inseparable pairs.

16:45-17:30 **Laurent Bienvenu**, *Solovay functions, randomness and triviality*.

In 1975, Solovay proved that there exists a computable upper bound F of the prefix-free Kolmogorov complexity function K , such that $F(x) = K(x)$ for infinitely many x . In this talk, we will consider the family of such functions, which we call "Solovay functions". After giving several ways to characterize these functions, we will show that they have deep connections with many central concepts in algorithmic randomness such as Martin-Löf randomness, K-triviality and lowness for Ω .



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Tuesday, July 6

10:00-10:45 **Russell Miller**, *Computable Fields and their Algebraic Closures*.

We present recent results about computable fields F and embeddings of F into its algebraic closure. We focus on fields which are algebraic over their prime subfields. The discussion begins with a review of Rabin's Theorem, and continues with more exact computability-theoretic comparisons of the set of irreducible polynomials in $F[X]$, the set of polynomials there with roots in F , and the different ways F can sit within computable presentations of its algebraic closure. We also examine this issue for noncomputable fields, considering the spectrum of the field F and its spectrum as a unary relation on the algebraic closure. The recent results presented are the work of Frolov, Kalimullin, Steiner, and the speaker, in various combinations.

11:00-11:45 **Selwyn Ng**, *Degrees of members of Π_1^0 classes*.

We will talk about some recent results regarding the c.e. degrees which can be represented in a Π_1^0 class. We show that given any c.e. degree a , there is a perfect Π_1^0 class P where every member of c.e. degree in P has degree a . We will also discuss the complexity of index sets which can be realized in this way, and some accompanying results. This is joint work with Barbara Csima and Rod Downey.

14:45-15:30 **Karen Lange**, *Computability of integer parts*.

An *integer part* I of a real closed field R is a discrete ordered subring containing 1 such that for all $r \in R$ there exists a unique $i \in I$ with $i \leq r < i + 1$. Mourgues and Ressayre showed that every real closed field R has an integer part. For a countable real closed field R , we showed that the integer part obtained by the procedure of Mourgues and Ressayre is $\Delta_{\omega\omega}^0(R)$. We would like to know whether there exists a construction that yields a computationally simpler integer part, perhaps one that is $\Delta_2^0(R)$. All integer parts are *Z-rings*, discretely ordered rings that have the euclidean algorithm for dividing by integers. By a result of Wilkie, any *Z-ring* can be extended to an integer part for *some* real closed field. We show that we can compute a maximal *Z-ring* I for any real closed field R that is $\Delta_2^0(R)$, and we then examine whether this I must serve as an integer part for R . We also show that certain subclasses of $\Delta_2^0(R)$ are not sufficient to produce integer parts for R . We construct a computable real closed field with no n -c.e. integer part for any $n \in \omega$. This is joint work with Paola D'Aquino and Julia Knight.

15:45-16:30 **Jennifer Chubb**, *Algorithmic properties of orderings on groups*.

A group is left-orderable (bi-orderable) when it admits a linear ordering of its elements that is left-invariant (bi-invariant) under the group action. The collections of left- or bi-orderings admitted by a group have been studied classically, perhaps most actively in the mid-20th century by Birkhoff, Clifford, Conrad, Fuchs, Mal'cev, B.H. Neumann, Šimbirova, and many others. More recently, the effective content of the theory of orderable groups has received attention, especially by Solomon beginning in the late 1990's. Here, we are interested in the spectrum of Turing degrees realized by the orderings admitted by a given group and in this talk will see conditions sufficing to ensure that a given computable group admits an ordering of every Turing degree. This work is joint with Mieczyslaw Dabkowski and Valentina Harizanov.