Definability of jump operator in the ω-enumeration degrees

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ω-enumeration reducibility and degrees

- For a given sequence $\mathcal{A} = \{A_k\}_{k<\omega}$ of sets of natural numbers, denote

$$J(\mathcal{A}) = \{deg_T(X) : A_k \leq_{c.e.} X_T^{(k)} \text{ uniformly in } k\}.$$

- $$\mathcal{A} \leq_\omega \mathcal{B} \iff J(\mathcal{B}) \subseteq J(\mathcal{A});$$
$$\mathcal{A} \equiv_\omega \mathcal{B} \iff J(\mathcal{B}) = J(\mathcal{A}).$$

- Given sequence of sets of natural numbers $\mathcal{A}$, set $deg_\omega(\mathcal{A}) = \{\mathcal{B} : \mathcal{B} \equiv_\omega \mathcal{A}\}$;
- Denote by $\mathcal{D}_\omega$ the set of all ω-enumeration degrees.

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$\mathcal{D}_\omega$ is an upper semi-lattice:

- partial order: $deg_\omega(\mathcal{A}) \leq deg_\omega(\mathcal{B}) \iff \mathcal{A} \leq_\omega \mathcal{B}$;
- least element: $\mathbf{0}_\omega = deg_\omega(\{\emptyset\}_{k<\omega})$;
- l.u.b.: $deg_\omega(\{A_k\}_{k<\omega}) \vee deg_\omega(\{B_k\}_{k<\omega}) = deg_\omega(\{A_k \oplus B_k\}_{k<\omega})$.

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jump operation, jump inversion

- Define $(A_0, A_1, A_2, \ldots, A_n, \ldots)' = (A_0' \oplus A_1, A_2, \ldots, A_n, \ldots)$;
- $\mathcal{A} <_\omega \mathcal{A}'$ and $\mathcal{A} \leq_\omega \mathcal{B} \Rightarrow \mathcal{A}' \leq_\omega \mathcal{B}'$;
- $deg_\omega(\mathcal{A})' = deg_\omega(\mathcal{A}')$;
  $\{\emptyset^{(k+1)}\}_{k<\omega} \in \mathbf{0}_\omega'$.
- least jump inversion:
  For each $\mathbf{a} \in \mathcal{D}_\omega$ above $\mathbf{0}_\omega^{(n)}$ there exists a least solution to the equation

$$\mathbf{x}^{(n)} = \mathbf{a}.$$

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Embedding of enumeration degrees. Definability

- $\kappa : \mathcal{D}_e \to \mathcal{D}_\omega$ defined by

$$\kappa(deg_e(A)) = deg_\omega(A, \emptyset, \ldots, \emptyset, \ldots)$$

  is an embedding which preserves:
  - the order: $\mathbf{x} \leq \mathbf{y} \Longrightarrow \kappa(\mathbf{x}) \leq \kappa(\mathbf{y})$,
  - the l.u.b. operation: $\kappa(\mathbf{x} \vee \mathbf{y}) = \kappa(\mathbf{x}) \vee \kappa(\mathbf{y})$,
  - and the jump: $\kappa(\mathbf{x}') = \kappa(\mathbf{x})'$;
- the range of κ is denoted by $\mathbf{D}_1$, $\mathbf{D}_1 = \kappa[\mathcal{D}_e]$;
- (Soskov, Ganchev) $\mathbf{D}_1$ is definable in $\mathcal{D}_\omega$ by a first-order formula in the language $\mathcal{L}(\leq, ')$;
- (Soskov, Ganchev) $Aut(\mathcal{D}_e) \cong Aut(\mathcal{D}'_\omega)$;

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A property of the least enumeration degree

- $\mathbf{0}_e$ is the only enumeration degree $\mathbf{x}$, such that:

$$(\forall \mathbf{y})[\mathbf{x} \vee \mathbf{y} \geq \mathbf{0}_e{}' \rightarrow \mathbf{y} \geq \mathbf{0}_e{}'].$$

- (Ganchev, M. Soskova) for every nonzero enumeration degree $\mathbf{u} \in \mathcal{D}_e$, $\mathbf{u}'$ is the greatest among the all least upper bounds $\mathbf{a} \vee \mathbf{b}$ of nontrivial $\mathcal{K}$-pairs $\{\mathbf{a}, \mathbf{b}\}$, such that $\mathbf{a} \leq_e \mathbf{u}$.

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Definability of the enumeration degrees

- What is the class of ω-enumeration degrees, defined by the above formula?
- $\widetilde{\mathbf{D}_1} = \{\mathbf{x} \in \mathbf{D}_\omega \mid (\exists \{X_k\}_{k<\omega} \in \mathbf{x})[X_0 = \emptyset]$;
- (Ganchev, S.): $\mathbf{0}'_\omega$ is first-order definable in $\mathcal{D}_\omega$;
- $\widetilde{\mathbf{D}_1}$ is first-order definable in $\mathcal{D}_\omega$;
- $\mu(\mathbf{a})$ is the least (ω-enumeration) degree $\mathbf{x}$, for which exists degree $\mathbf{y} \in \widetilde{\mathbf{D}_1}$ s. t. $\mathbf{x} \vee \mathbf{y} = \mathbf{a}$.;
- $\{A_k\}_{k<\omega} \in \mathbf{a} \Longrightarrow (A_0, \emptyset, \ldots, \emptyset, \ldots) \in \mu(\mathbf{a})$;
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Definability of the jump

- Definability of the jump operation is equivalent to the definability of $\mathbf{D}_1$
- 1. (Kalimullin) The jump is definable in the substructure $(\mathbf{D}_1, \leq)$;
  2. (Ganchev) for each $\mathbf{a} \in \mathcal{D}_\omega$, there are $\mathbf{x}, \mathbf{y} \in \mathbf{D}_1$, s. t.

$$\mathbf{a} = \mathbf{x} \wedge \mathbf{y};$$

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