The Electrostatic Properties of Zeros of Exceptional Polynomials

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Exceptional orthogonal polynomials were introduced recently by D. Gómez-Ullate, N. Kamran and R. Milson. All known families of exceptional polynomials are orthogonal polynomials with respect to a weight of the form

$$w(x) = \frac{w_0(x)}{P^2(x)},$$

where $P(x)$ is a polynomial. The classical counterpart of the exceptional polynomials in question are orthogonal with respect to w_0 on an interval I and P has no zeros in I. We call as "regular zeros", the zeros in I and as "exceptional zeros" the zeros out of I. The location of zeros of exceptional Laguerre and Jacobi polynomials are described by D. Gómez-Ullate, F. Marcellán and R. Milson, of exceptional Hermite polynomials by A. Kuijlaars and R. Milson.

We examine the electrostatic properties of exceptional and regular zeros of exceptional Laguerre and Jacobi polynomials. Since there is a close connection between the electrostatic properties of the zeros and the stability of interpolation on the system of zeros, we can deduce an Egerváry-Turán type result as well. The limit of the energy on the regular zeros is also investigated.

Since the exceptional zeros of exceptional Hermite polynomials are complex, the situation here is different. We localize the eigenvalues of the Hessian in general cases. In some special arrangements we can state more precise result on behavior of the energy function.