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Personalization of secondary school mathematics education through the use of modern information technologies

Ph. D. Thesis

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ABBREVIATIONS

NCTM	National Council of Teachers of Mathematics
ICMI	International Commission for Mathematical Instruction
ITS	Intelligence Tutoring Systems
AHS	Adaptive Hypermedia Systems
AITS	Adaptive Intelligent Tutoring System
INIMA	Institute of Applied Informatics and Mathematics
UNDP	United Nations Development Program
DigCompEdu	Digital Competence of Educators
SELFIE	Self-reflection on Effective Learning by Fostering the use of Innovative EducationalTechnologies

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INTRODUCTION

The beauty of personalized learning is that even the smallest changes can make the biggest difference

Pernille Ripp

Pre-introduction

During a lesson about personalized math teaching with professional Master's degree students who will become math teachers in the future, I asked the question: Did your math teachers personalize the lesson during your pre-university education?

Among the many answers, one of them was very interesting and has remained in my mind to this day. It was the story of the best and most passionate student in the course.

The student said: "*Ever since I was in the fifth and sixth grade, my parents have signed me to take private math lessons privately, in parallel with what I took in school, with a very good math teacher.*

Although I studied a lot and had the daily support of the private tutor, who personalized my math lesson every day, I never had a very good math education until the seventh grade when our new math teacher turned the class into "a mathematical market", where mathematical thoughts flourished, where each of us felt free to express his mathematical thought by bringing his point of view to various mathematical issues.

Even today, I can't understand how she found the time while instructing students with learning difficulties, with a question or just a clue, to stimulate the mathematical thinking of talented students, organized debates and discussions on mathematical issues, encouraged and supported not only our achievements, but she also used our mistakes to build solid mathematical knowledge.

My teacher, - the student continued,*often gave us tasks that promoted logical and critical reasoning and was careful that their solution was made using extensive mathematical knowledge. It created the conditions for us to work in groups to generate ideas and express them with mathematical competence.*

One aspect of her teaching that I fanatically memorize,was the eloquent and carefully selected questions that she provided. Their answers gradually constructed new mathematical concepts. She was there to support each one of us, and it seemed to each one of usthat and felt it.

Her passion and creativity in class, supported only by 6 computers, a printer, a colour photocopier, our calculators, and her laptop, not only deepened my mathematical thinking but revolutionized my thinking about mathematics as a scientific field and motivated me to continue my studies as a math teacher. I owe her everything I am todayand I hope that one day someone else will be motivated by me the same way"- closed her discussionwith my student.

1 RELEVANCE OF THE PROBLEM

Today's generation of students faces everyday personalized processes in almost all aspects of daily life. This personalization should be accompanied by the personalization of their learning too.

The many changes that are taking place day by day around the world bring attention to the education policymakers' review of the need for a periodical review of the policiesin terms of learning mathematics, turning them into a necessity andincreasing the quality of mathematical thinking and, to solve as much as possible actual practical problemsof our daily life.

Assessment of mathematical thinking has been important in all PISA tests, but it has been its focus in 2003, and 2012 and has been returned to PISA 2022, to identify the change it has undergone as a result of all the changes that have taken place in the education policies of different European countries.

In recent times, the digitization of many aspects of life, the ubiquity of data for making personal decisions involving initially education and career planning, and, later in life, health and investments, as well as major societal challenges to address areas such as climate change, governmental debt, population growth, the spread of pandemic diseases and the globalizing economy, have reshaped what it means to be mathematically competent and to be well equipped to participate as a thoughtful, engaged, and reflective citizen in the 21st century. (OECD, 2018)

Relying on the trend of personalization in all spheres of life, the century we are living in came with a completely different approach from that of the 20th century in terms of educational policies. The characteristics of education of this century are:

1. Personalized learning;
2. Equality, diversity, and inclusiveness;
3. Learning through doing;
4. Changed teacher roles;
5. Relationships with the community;
6. Technology;
7. Professionalization of the teacher. (Bolstad, et al., 2012).

There is no coincidence that the first point of these key characteristics is the personalization of learning. It is placed in coherence with personalization in all aspects of life, which undoubtedly will be intertwined and strongly supported by other important components like ICTs.

Since the teaching of mathematics is the basis of the teaching of all sciencesthe personalization of the teaching of mathematics takes on a special role in the educational system as a whole.

Children of all ages have developed a certain intellectual potential, which is also under the pressure of a very complex digital world. But on the other hand, like any other human being, they have different development of theirsenses.It should be the responsibility of all math teachers to adapt their strategy of teaching, accordingto the personal skills of their students.

Chris Drew (Drew, 2019) proposed a theory of learning modalities, which classifies the learning skills of students into four main groups: visual, auditory, kinesthetic, and sensory, given the fact that the different students from one may have class have different aptitudes and skills, math teachers should be prepared to use a variety of modalities to suit their needs.

Since students' skills and habits of intelligence are very diverse, as well as the fact that the methods and techniques of teaching mathematics are very diverse, it is necessary to adapt them to each other and to treat them carefully with modern ICT-s tools care.

Based on the arguments above, in general terms, a personalized learning model can be expressed as follows in Figure 1.

Personalized learning = Student's need + Learning modality + ICT's tools support

Figure 1 Personalized learning model

Unlike older teaching models, where all students learn the same content, the personalization of math teaching requires a differentiation of the content and its difficulty.

That's because the lack of personalization is focusing teachers on the average level and that way they tend to hold back talented students and lose those with lower math skills. By looking at a wide range of modalities, techniques, and strategies for personalized learning, we notice that implementing ICT in teaching usually increases the opportunities for a better math lesson. AsCuoros (Cuoros, 2021)*says, technology is not just a tool. It can give students a voice they may not have had before.*

Using personalization,the students are given a large set of strategies, and their overall school experience starts to respond to their talents and their personal needs and interests.

Regarding (Izmestiev, 2012) says: *Technology enables personalized instruction through the adjustment of learning strategies and content for each student. This approach has become very popular in e-Learning, but can also be applied in the classroom, when the selected distribution of digital content becomes part of personalized instruction.*

Personalized learning in mathematics through ICT is a process that requires a balanced and well-studied combination of many components which range from recognizing students' skills and needs, technological support, digital competencies, student's learning style, and most importantly the skills of the teacher to present each of these components the right time.

This **PhDthesis is a model of a process** that requires a deep understanding of the skills, needs, and talents of the students to be combined with many other psycho-pedagogical factors,on one hand,and the continuously updated implementation of educational technology innovations for personalization of teaching and learningmath,on another.

The thesisgives a model that can be followed to improve the effectiveness of personalized teaching/learning of mathematics through ICTs. It aims to introduce positive cases for the use of ICT in teaching mathematics, based on continuous training of teachers, for the use of educational technological platforms, as well as other components of the multidimensional teaching process.

In particular, teachers' training should be considered a very serious task for all actors dealing with educational policies in Albania.

2 OBJECT AND SUBJECT OF THE RESEARCH

The **objects** of study in this doctoral thesis are:

1. The technological base of Albanian schoolsis a key tool to support the personalization of teaching in general and mathematics in particular.
2. Mathematics teachers in high schools in Albania;
3. Students in high schools in Albania.

The **subjects** of the study are:

1. The required conditions for mathematics teachers in Albania to provide personalized teaching with the support of ICT tools;
2. The digital competencies of math teachers in Albania;
3. The sensitivity of Albanian high school students to personalized ICT-supported teaching/learning of mathematics and the learning outcomes.

The requirements of school principals the educational policy-making for modern ICTs infrastructure in schools and the development of a model for teacher training (or recommendations) are the key factors for the research.

3 RESEARCH QUESTIONS

The research questions of this study are:

1. Is the existing technological base in the Albanian schools and the digital competencies of the Albanian teachers good enough for personalization with ICT-supported mathematics education?
2. How important is the continuous training of teachers in terms of learning modalities and support of ICTs tools, for active contemporary teaching in mathematics?
3. Is the personalization of math learning through ICT increasing students' performance, as well as their motivation and self-confidence to deal with math?

4 HYPOTHESIS

The hypotheses of this study are:

1. The insufficiency of ICT tools in Albanian schools is a barrier to the wide adoption of educational technology for mathematics learning personalization;
2. The lack of digital skills and competencies of Albanian teachers slows down the process of personalization of math teaching in Albanian schools;
3. The understanding of the students learning styles and the use of ICTs tools in mathematics teaching is a necessity for personalized learning, in recent years;
4. The use of ICTs tools in teaching/learningmathematics leads to higher gradesand better self-esteem and motivationfor students.

5 OBJECTIVES

The **objectives** of the research are:

1. To collect data to analyze the real situation of the technological base in the function of teaching that exists in our schools, and in parallel to investigate the perception of mathematics teachers in secondary school on the role of ICTs tools in personalizing mathematics teaching.
2. To Train a group of secondary school mathematics teachers to improve their skills and competences firstly in the factors that lead to the personalization of teaching and secondly in the efficient use of SmartBoard, GeoGebra and Kahoot!, to personalize the teaching of mathematics, bringing a model to be followed by training agencies in Albania.

3. To measure the extent to which ICT tools influence, to support the personalization of mathematics teaching, the development of mathematical competencies (student outcomes) as well as their motivation to learn mathematics.

6 TASKS

To achieve the purpose of the dissertation, the research was based on the following tasks:

1. Analysisof the ICT infrastructural base in the Albanian educational system, in the Albanian schools, as well as the digital skills and competencies that the Albanian mathematics teachers have, to apply the ICT tools in teaching.
2. Analysis of data regarding frequency, information, and diversity of use of ICT to personalize math learning in Albania.
    a. To explore the types of ICTs, used by mathematics teachers / available for a mathematics teaching.
    b. To investigate the readiness of mathematics teachers to use ICTs in the classroom
    c. To investigate the interestthey have in being trained in the use of ICT in teachingmathematics.
    d. To investigate the personal experience with the use of ICT in the classroom.
    e. To identify the difficulties encountered by mathematics teachers in Albania in using ICT in teaching mathematics.
3. Analysis of the role the training of mathematics teachers has, concerning students' learning styles and adaptation to ICT tools for them
4. Training of a group of mathematics teachers on the use of some platforms and technological tools to personalize the mathematics teacher.
    a. To clarify the role that teachers' knowledge of learning styles plays in adapting teaching strategies to learning styles.
    b. To give examples of the support that ICT tools provide for each of the learning styles in mathematics learning.
    c. To train teachers on using some math education platforms to personalize math learning
    d. To discuss the effectiveness of the training and explore their willingness to implement ICT tools in teaching.
5. Analysis of the role, played by the use of ICT tools for the personalization of teaching and learning of mathematics, as well as the reactions of teachers and students involved in the study.
6. Case study on the implementation of ICTs tools (SmartBoard, GeoGebra, Kahoot!) in 3 schools to personalize the teaching/learning of mathematics.
    a. Analysing results ofteaching/learning mathematics personalization.
    b. Analysing the feedback of the teachers and students who underwent the experiment.

Case study 1: Personalized teaching and learning mathematics through problem-solving, supported by SmartBoard.

Case study 2: The efficiency of dynamic GeoGebra software, in giving differentiated instructions based on student levels, on treatment concept of functions, in the eighth grade

Case study 3: The impact of learning games on the approach of secondaryschool students towards the subject of mathematics and the development students of computational thinking.

7 RESEARCH METHODS AND TOOLS

1. **Analysis of data regarding frequency, information, and diversity of use of ICT to personalize math learning**

This studyfirstly has made a review of the regulatory framework, to determine the conditions in Albania for offering personalized teaching in math through ICT tools.

In function of the defined objectives and the verification of the hypotheses, the following study methodology was followed.

For investigating infrastructure has been made a review of a regulatory framework based on the ordinance, reports, and government strategies.

For investigating how the teachers use the ICTs in teaching math has been made a survey, focused on their ICTs skills, attitude to the role of ICTs in math education, perception about the role of personalized teaching, experience in the use of ICTs in providing personalized teaching

For this investigation were used statistical methods with the target group method. The data collected by the target group (mathematics teachers in the 4 Regional Education Directorates (RED) in Albania), was enriched by discussions and interviews with teachers in the district of Elbasan city. From the discussions with them, data was collected on how they see the personalization of learning mathematics, and if they include the use of technology in their approaches to the personalization of learning.

So, as a start, we will shed light on the access to ICT tools that mathematics teachers have in secondary schools in Albania, and in parallel, we will study whether they are ready to use ICT tools in teaching.

2. **Training of a group of mathematics teachers on the use of some platforms and technological tools to personalize the mathematics teaching**

The training of math teachers was closed with a questionnaire developed regarding the effectiveness of the training. The questionnaire aimed to investigate:

a. The level of knowledge they possessed about the support that ICT provides, according to learning styles to personalize mathematics teaching/learning.
b. Do they build a Personal Learning Plan (PLP) in their day-to-day work to refer to it as a basic document for personalizing math learning? If they build it, do they refer to it in their daily work?
c. What are the challenges and problems that prevent them from personalizing math teaching?
d. How much do they appreciate the support that ICT tools provide for conducting personalized learning in mathematics?

The short questionnaire, at the end of the training, was made to gather the necessary information to investigate the third hypothesis as well as to observe the above indicators.

The training was finalized with the voluntary selection of two teachers who would implement in their schools, personalized teaching based on ICT tools.

3. Case study on the implementation of ICT (SmartBoard, GeoGebra, Kahoot!) in 3 schools to support personalizing teaching/learning of mathematics.

This part of the study brings results, facts, and experiences, from the implementation of several platforms and technological tools in the teaching of mathematics in 3 schools, where teachers manage to realize personalized teaching in mathematics, as well as will collect data on the feedback of students who underwent the experiment. At this stage have been used experimental and statistical methods.

Case study 1: Use of SmartBoard to support problem-solving, for students with different learning styles, seventh graders.

In this case, the study examines the empowerment of logical and critical reasoning problem-solving, through active teaching methodologies, supported by SmartBoard, to personalize mathematics learning.

1. **Independent variable**: Problem-solving (according to the steps defined by George Polya), with pencil, blackboard, paper, and SmartBoard.
2. **Dependent variables**: the results of the third quarter as well as the improvement of problem-solving techniques.

In this experiment the whole process followed this path:

- **Phase 1**: Analysis of the situation of the indicator of dependent variables for the second quarter.
- **Phase 2:** The dependent variables is influenced by the independent ones.
- **Phase 3:**The indicators of the dependent variables are measured through math test for students. Analysis of results are used to investigate whether the indicators have changed and how.

Case study 2 explores the use of GeoGebra in terms of visualization, demonstration, group work, or mathematical modelling of problems, related to the concept of function, and increasing the level of personalization, motivation, and identification of talent in learning mathematics.

The process followsthe path:

1. **Independent variable:** teaching method (traditional with pencil, blackboard, GeoGebra Software)
2. **Dependent variable:**tracked by apost-test of achievements that include:
    a. Capability for discovering laws, using the meaning of algebraic functions and symbols to model mathematical relationships and situations.
    b. Problem-solving skills, related to functions.
    c. Skills for description and mathematical modelling of problem situations, based on "size" concept, with a context from real life.
    d. Skills for formulating conjecture and judgment of conjectures.
    e. Skills for planning and structuring mathematical arguments for the conclusions found.

f. Capability to create variety of representations of a problem and its mathematical model – by drawings, by the use of ICT, algebraic mathematical concepts, and graphics.
g. Capability to link new mathematical concepts and models to those previously acquired by mathematics and other fields and understanding their formation.

The process followsthe path:

- **Phase 1**: Analysis of the level of knowledge on the topic "Functions"
- **Phase 2:** The dependent variables was influenced by the independent ones
- **Phase 3:**The indicators of the dependent variables are measured through math test for students. Analysis of results are used to investigate whether the indicators have changed and how.

Case study 3 aimed to investigate the effect of learning games on the development of logical, critical, and computational thinkingof students at grades 5. –9. in the secondary school "Imelda Lambertini" in Elbasan, Albania.

This case study will investigate the role that the usage of the Kahoot!, plays for:

a. Students' motivation;
b. Self-esteem and cooperation of students;
c. Strengthen positive attitudes toward learning mathematics;
d. Logical thinking development;
e. Computational thinking development.

Independent variables: critical and computational thinking in problem-solving and testing knowledge with pencil and paper and Kahoot!

Dependent variables: results achieved, as well as motivation and emotional behaviour of the students.

The process followsthe path:

a. Phase 1: "Contest of challenges in mathematics". In this phase all students of the selected classes participate, regardless of their level of mathematical knowledge and skills, and each of them chooses another student to challenge him. The challenge is to present solutions of the problems, carefully chosen by the math teachers, according to the computational thinking scheme.
b. Phase 2: "Evaluation of students' achievement through the Kahoot! application".The dependent variables were put under the influence of independent ones, and tests were performed with pencil and paper and using the Kahoot! application.
c. Phase 3: The indicators of the dependent variables are analyzed to investigate whether they have changed and how.

So in all case studies, it was measured, manipulated, and observed what benefits bring the amplification of active teaching methods with the support of ICT tools tools, to personalize mathematics learning.

The analysis of the questionnaires, performed by the school students, who underwent the case studies, showed that the personalization of mathematics learning, supported by educational technological tools increases the interest in learning mathematics and motivates the students.

Also, the oral interviews with the teachers of the experimental groups brought a positive reaction to their new experience. They saw the proven experience as a fresh start for a continuation of the process of personalizing their teaching, supported by ICTs tools.

The study is **development oriented**.

Oriented,because it defines the direction, goals, and areas, and will serve as a good practice to be implemented and used further in other schools. It is **developmental** because the study brings conclusions and makes recommendations through which the situation can change by progressing towards the personalization of mathematics learning through technology.

8 STRUCTURE AND THE CONTENT OF THE PHD. THESIS

This doctoral thesis is conceived according to the following structure:

After an introduction it follows **CHAPTER 1.Overview of the role of ICT tools in teaching mathematics. Approachesto their applicationsin the Albanian context.**, which presents a literature review on the role that ICT tools playto support personalizing mathematics learning.

It presents a systematic analysis of the evolution of the mathematics learning process, from pencil and paper teaching to modern mathematics teaching.The main focus is how the evolution of technological tools supports active mathematics teaching. In the second part, the chapter presents a national context review in terms of the regulatory framework and the school ICTs infrastructure in Albania.

CHAPTER 2. Personalized learning in mathematics,presents the general treatment of personalized learning and analysesthe mathematical context of the four main components of successful personalized learning:

- Reflecting on students and setting their goals;
- Targeted instruction;
- Flexible path and pace;
- Collaboration and creativity.

The full framework of the chapter concludes with some instructions on how the personalized math lesson can be accomplished and which tasks the teacher must accomplish.

CHAPTER 3.Use of ICT in teaching mathematics in Albania: state-of-affaireaims to shed light onAnanalysis and interpretation of the questionnaire developed with the target group of teachers of mathematics in secondary school about these issues:

- **a.** The types of ICT tools that teachers have at their disposal for teaching in the schools where they teach.
- **b.** Their perception of the use of ICTs tools in mathematics learning.
- **c.** The interest they have in being trained in the use of ICTs tools in teaching.
- **d.** Personal experiences with the use of ICTs tools in the classroom.
- **e.** Difficulties encountered by mathematics teachers in Albania in using ICTs tools in teaching mathematics.

CHAPTER 4.Training teachers to provide personalized math education, supported by ICT tools, presents the aspects of training math teachers in Elbasan, Albania about personalized learning in mathematics based on the learning styles of students and the ICT support, as well as the opinions of some teachers on these issues.

CHAPTER 5. Validation of teachers' training on personalized math teaching, supported by ICTs, analyses 3 experiments performed. Their successful implementation in these schools, by teachers who volunteered after the training, confirms the effectiveness of the training and brings facts that personalized teaching of mathematics supported by ICTs tools brings not only higher results in students' mathematical thinking, but also increases their motivation and self-esteem.

CHAPTER 6.Influence of ICT-supported personalized learning of mathematics on the motivation to learn mathematics, addresses therole played by the use of educational technologies in teaching mathematics to motivate students and increase their self-esteem. The results were based on the analysis of a questionnaire made by students who underwent the experiment.

Conclusionspresents concrete conclusions and recommendations, starting from the design, implementation, and transparency of the implementation of educational policies. It discusses the function of teaching and suggests continuous trainings of teachers to amplify personalized teaching of mathematics with the right ICT tools.

In this chapter are made some recommendations addressed to the faculties that prepare mathematics teachers,for the ambitious development of their digital competencies, to renew the subject curriculum "Didactic tools in teaching mathematics", with software, ed-tech didactic games, for the teaching of mathematics. For older teachers, policies are implemented that enable their continuous training on the use of ICT tools in support of personalized learning and teaching.

The author's reference is a summary of scientific contributions and presents publications related to the dissertation.

Appendix 1, presents the questionnaire about personalized learning in mathematics and the role that ICTs tools play developed with 210 teachers of mathematics in Albania, to aim to collect data on the frequency, quality, and variety of use of ICTs to personalize the teaching of mathematics, which in general means the design, development, and implementation of teaching strategies and techniques taking into account the skills and intelligence of each.

Appendix 2, presents **the questionnaire for teachers regarding the effectiveness of training**

The training was conducted with 42 teachers from the district of Elbasan as well as the teachers of the "Vinçens Prendushi" secondary school in Durrës, which ended with a questionnaire to measure the effectiveness of the training.

Appendix 3, presents **a test for the identification of students' learning styles.**

Appendix 4, presents **the student questionnaire on the use of ICT tools in personalized learning and teaching in mathematics.**

Appendix 5, presents **the agenda for training days in March 2021.**

CHAPTER 1.OVERVIEW OF THE ROLE OF ICT TOOLS IN TEACHING MATHEMATICS. APPROACHESTO THEIR APPLICATIONSIN THE ALBANIAN CONTEXT.

Technology can become the "wings" that will allow the educational world to fly farther and faster, than ever before - if we will allow it.

Jenny Arledge

This chapter aims to clarify the following issues:

- What is the role that ICTs play in the modernization of education in general and in mathematics in particular?
- What were the parallel evolutions that mathematicians and ICT thinkers recognized and how did they influence each other's perfection?
- What are the evolution and challenges of ICTs in Albania?
- Does the infrastructure comply with the regulatory framework on teacher competencies in Albania, and how close are they to the UNESCO ICTs competencies framework for teachers?
- What are the benefits that the use of ICTs offers in improving the teaching and learning of mathematics and how ambitious are the policies of the Albanian government to introduce this radical change in the mathematics of secondary school education?
- What are the approaches of the regulatory framework to the competency-based mathematics learning curriculum, and has its implementation led to changes in the results of our students at PISA?

1 THE ROLE OF ICTS IN MATHEMATICS EDUCATION - OVERVIEW

In a tech-savvy world, tech-savvy education is a must. When we talk about education in general, mathematical education is undoubtedly the priority of all other types of education. Ersilia Vaudo Scarpetta, (an Italian astrophysicist, in the position of Special Adviser on Strategic Evolution to the European Agency for Reconstruction and Development) provides brilliant reasoning on the role of mathematics in human society. *Mathematics will save our children and democracy*, and further explains: *The inclusion of all in quantitative language means strengthening the skills of analysis and critical spirit and the ability to give children who will become adults, the means to exercise the city. Mathematics serves to preserve democracy.* (Sarno, 2021)

This statement is in full sync with the educational policies of the European Union, which since 2009, has valued mathematical competencies as the main skills for educating active citizens, fulfilled from a personal and professional point of view, and has defined a task of how this can be achieved (European Commission, Directorate-General for Education, Youth, Sport and Culture, 2019).

However, it is a fact that the interest of young people to deal with mathematics is declining more and more. Although various European countries have often developed ambitious policies for mathematics learning, in the international PISA survey, which is conducted for this purpose, the results of young people in mathematical literacy are not satisfactory.

In PISA tests the focus of the test is the mathematical competence that a 15-year-old should possess. This competence has been defined since 2003, as *The ability of an individual to identify and* *understand the role that mathematics plays in the world, to formulate well-founded judgments, and to use and approach mathematics in ways that meet life's needs that individual as a constructive, interested and thoughtful citizen.* (Lemke, et al., 2004)

A look at PISA's policies on mathematical reasoning (deductive and inductive) since 2003, re-researched in 2012, and brought to attention in 2021, reveals that it relies on three important links:

a. Formulate
b. Employ
c. Interpretation and Evaluate

It is important to note that the definition of mathematical literacy not only focuses on the use of mathematics to solve real-world problems but also identifies mathematical reasoning as a core aspect of being mathematically literate. (OECD, 2018)

In 2021, unlike the other two years, the emphasis is on the synergy and reciprocal relationship between mathematical and computational thinking. C*omputational thinking complements and combines mathematical and engineering thinking. Computer science inherently draws on mathematical thinking, given that, like all sciences, its formal foundations rest on mathematics* (Wing, Computational Thinking, 2006).

The role of technology in mathematics education is increasingly emphasized in mathematics education policies around the world, but this role should not be exaggerated.

It is very important not to deviate from the focus of mathematics education which is:Mathematical learning, to "live" through mathematics, to make mathematics the light that illuminates and solves our problems that start from the simple ones of an algorithmic nature and go to the more complex ones that require logical analysis.

This focus should be fanatically maintained by mathematics teachers and education policymakers, as by wanting to reinforce mathematics education through technology we can deviate from its primary purpose.

For this reason, the National Council of Teachers of Mathematics (NCTM) emphasizes the strategic use of technology in teaching and learning mathematics, which means *the use of digital and physical tools by students and teachers in ways that are thought-provoking and carefully defined times, so that technology skills enhance the way students and educators learn, experience, communicate and do the math.* (National Council of Teachers of Mathematics, 2015)

Various researchers who have studied the role of technology in mathematics education have shed light on the various aspects and roles that technology plays in mathematical teaching and learning. The study (Gadanidis & Geiger, 2010)states that*changes in technology and the growing influence of social learning theories on mathematical education research, which acknowledge the role of social interaction in learning, imply that it is necessary to reconsider what it means to learn and do the math, seeing math teaching as an activity performed rather than passively acquired.*

This modernized view of learning mathematics has been accompanied by educational policies structured in content and form around the world, always emphasizing the empowering role that technology plays in teaching mathematics.

Giant developments in educational technology in the field of mathematics make the time when the supporting technology of mathematics learning revolved around the trio: calculator - computer - the internet. Day by day the variety of software and platforms supporting math teaching and learning is expanding.

In the complex world of mathematical education that faces a variety of challenges, technology aims to enter as modern support, it aims to facilitate the whole process of writing mathematics by resizing it. The importance of using ICT in every area of life, but in that of education, in particular, comes the *fundamental principle of the global rationalization of human intellectual activity through the use of information and communication technologies (hereinafter ICT). The ultimate goals of informatization of education are to provide a qualitatively new model for training future members of the information society, for whom active mastery of knowledge, flexible changes in their functions in work, the ability for human communication, creative thinking, and planetary consciousness will become a vital necessity.* (Mamurakhon, 2020)

It is clear that this process is not a process that lasts a lifetime, it must be a process of positive change, with clear ideas for achieving the final goals.

In the article of 2015 (Skryabin, Zhang, Liu, & Zhang, 2015), the role that plays not only the individual development of ICT but also the national one of ICT is evidenced. *According to the findings of this study, the national ICT development level is a significant positive predictor for individual academic performance in all three subjects for both 4th-grade and 8th-grade students, while the national economic development level was controlled for.*

Regarding the benefits of using ICT in teaching, and even in personalizing the math learner, the article (Drigas & Karyotaki, 2014) states that: *Learning technologies are an indispensable tool for students' cognitive improvement and assessment. ICTs in coordination with a concrete pedagogical framework may provide students and teachers with flexible, engaging, cost-effective, and above all, personalized learning experiences, which focus on the adoption of 21st-century cognitive skills into the actual learning process. Such Higher Order Thinking Skills (HOTS) entail critical thinking, problem-solving, independent inquiry, creativity, communication, collaboration, and digital literacy. Therefore, technologically-supported educational environments aim at self-regulated and inquisitive, constructivist, knowledge building rather than knowledge accumulation.*

The education of today's students, not only in mathematics, but globally, is not only a product of family, social, and school education, but also a product of the powerful access they have to social networks through technological means. For the latter, it is very important to orient them in the right way.

Talking about access to technology all our students have considering personal tablets or mobile phones did not necessarily have to expect higher results in mathematics.

Maybe this can have the opposite effect. Thus in a german study on this phenomenon, it was found that *students who owned a mobile phone, a computer/tablet as well as those who used computers/tablets "every or almost every day" at home, at school, and other places for schoolwork, achieved lower mathematics scores than their counterparts. While the shared use of a computer/tablet and an internet connection positively and significantly predict student performances* (Saal, Graham, & van Ryneveld, 2020).

Orienting students on math education software is very important.Regarding the role played by mathematical software, in the teaching of mathematics, various researchers have brought their opinions about the studies done. *Educational software offers students a much better visualization,* *dynamism, and opportunity for independent research and deepening their knowledge by searching the Internet and repeating the operations in software usage to better master the subject matter* (Pacemska, 2012).

For the successful integration of ICT, the role of the teacher is critical, because it is the teacher who decides when, where, how, and who will use ICT (Jackson, 2017).

The teacher should be the organizer of the whole process of orienting the students towards the software or technological platform that he thinks is most productive to personalize the teaching of mathematics. But for teachers to know where and how to orient their students throughout this process, they first have to be self-reliant to enhance their digital skills and competencies. Their ongoing training to keep up with tech developments is a key factor.

In a study by Drigas & Karyotaki, (Drigas & Karyotaki, 2014), it was found that community partnership programs to train math teachers to give a very good effect. *The results of this study indicate that there are differences in the ability of teachers in ICT before getting training and after getting training. The results of the side Wilcoxon rank-sum test show that the ability of teachers in the ICT field after being given training is better than before being given training. So it can be concluded that the Community Partnership program can effectively improve the ability of teachers to utilize ICT in mathematics learning.*

A very important role is also played by the motivation of teachers who integrate technological platforms into mathematics teaching. *The integration of ICT in education and their use of ineffective didactic practices are favoured when teachers' motivation increases.* (Méndez Coca, Méndez Coca, Anguita Acero, & Suárez Llevat, 2019)

Regarding the role and importance of using ICT in teaching mathematics, the results showed that *mathematical thinking and problem-solving evolve as students engage with ICT activities and learn cooperatively.* (Hashem & Arman, 2013)

On the other hand, the use of ICT in teaching mathematics requires good management of the learning process by finding the right technological resources and their use.

The professionalism of math teachers is the best weapon to solve this problem. *The teachers are the ones who should personalize the content towards the specific student and the didactical situation in question. Therefore, selection, adaptation, and combination of resources are among the major teacher's tasks* (Lokar, Luksic, & Horvat, 2012).

2 AN OVERVIEW OF THE DEVELOPMENT OF TRINOMA: THE HUMAN MIND –MATHEMATICS – COMPUTERS

The teaching of mathematics has undergone fundamental changes over time and at different eras of human thought. It has always played a central role in the education of human societies, although its positioning in education as a whole has often been the subject of debate in various academic societies, as well as in the community at large. Regardless of the way this science has been treated from ancient times to the present day, no matter what topics have been the subject of mathematical study at different times, whatever the tools and techniques used, one thing is clear: the teaching of mathematics existed millennia before the invention of computers.

Let us mention here the discovery of the letter from the Chinese, and then the discovery of the printing press by Gutenberg (1450), while in 1700 we have the use of blackboard and chalk as the first two elements in teaching, all to come to the third element, the book (in 1800).

In the late 1940s, we entered the beginnings of the **audiovisual era**, which undoubtedly had its impact on teaching. And to be honest, the development of mathematical thinking owes very much to these tools that have existed to make mathematics more tangible.Nowadays the "Trinoma" composed of the **human mind - mathematics – computers** (Figure 2**Error! Reference source not found.**) are becoming very common.

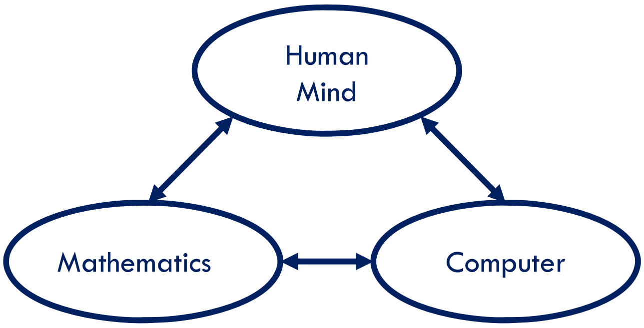


Figure 2. Trinoma the human mind - mathematics – computers

Referring to the scheme as well as the origin of the word "computer", the existence of this Trinoma is easily explained. The word *computer* was used long before electronic computers appeared commercially. The term was first used in 1613 to mean a person (a human mind) who performs mathematical calculations. The *human-computer* follows fixed rules and has no right to change any of them in any way. (Oxford Languages, 2008)

Meanwhile, we are talking about electronic computers that have invaded the modern world, giving innumerable impulses to the development of human societies through technological development. This influence is also evident in mathematics teaching.

Mathematics teachers around the world make every effort to implement it in mathematics teaching, to make the teaching process more attractive and efficient. Their day-to-day work relies not only on chalk, books, and blackboards but also on the immense help that computers, especially software, are providing in the teaching process.

Learning mathematics is a daily challenge for anyone who is part of the teaching process, though not for the whole society at large because all areas of life are closely linked to mathematics learning. There is a belief that students hate math, but as Larry Martinek put it, *Children do not hate math. What they hate is being confused, intimidated, and embarrassed by math. With understanding comes passion and with passion comes growth - a treasure is unlocked.* (Martinek, 2022).

Therefore, throughout the centuries, finding ways to make mathematics teaching attractive has been the focus of teachers of all times. Regardless of the various methods, we are aware of, we often fail to do this well. Greenlight has come on with the introduction of technology into the process of teaching mathematics. It is about implementing ICT in this process.

Nowadays, in addition to implementing new student-centred methods, there is a continuing effort to implement ICT in teaching mathematics, meaning Technology will never replace great teachers, but technology in the hands of a great teacher can be transformational. (Couros, 2015)

Throughout the world, ICT is changing the face of modern education in general and mathematics in particular. Although there are still groups of sceptics, ICT is taking an important place in what is today known as modern education. It is required to maintain a balance between the gallant development of ICT in society, its implementation in schools, and its use in the teaching process.

Even the policies of different states have made it clear what Jenny Arledge has pointed out, technology can become the "wings" that will allow the education world to fly farther and faster than ever before - if we allow it. But besides the need and the goodwill to make this integration, different countries, including Albania, face great challenges. These challenges are different, ranging from lack of infrastructure, access to technology to the entire population, lack of special programs in the curricula of universities that relate to the well-being and use of technological resources in the teaching of mathematics, and the teachers' lack of ability to use technology effectively in classrooms. The latter is undoubtedly the main challenge of this process. If we were to translate all the challenges mentioned above in the language of the variables, they should be the variables that the governments of the respective countries should consider when formulating education policies. The sooner these policies are implemented, the earlier the pedagogy as a whole will be transformed, the more inclusive the education will be, and the greater the interaction between the community and the school. Only in this way will the children of today's schools be the citizens of tomorrow, who will face the unstoppable development of technology.

3 ICT IMPLEMENTATION IN TEACHING AND LEARNING MATHEMATICS

3.1 Historical overview of the ICT implementation in teaching and learning mathematics

Implementation of ICT in teaching mathematics could not begin immediately with the introduction of computers into our lives. In its beginnings the computer was the*property* of programmers who could not go beyond simple programming with a language directed at the world of numbers, so they were used to write numerical algorithms, as very short programs in BASIC.

About 50 years ago, the next step came in the algorithmic style, with the appearance of the "Logo" programming language on various personal computers. The Logo is an educational programming language designed in 1967 by Wally Feurzeig, Seymour Papert, and Cynthia Solomon (Abelson, Goodman, & Rudolph, 2016). The Logo is not an acronym: the name was coined by Feurzeig while he was at Bolt, Beranek, and Newman and derives from the Greek logos, meaning word or thought. (Paul, 2013).

The **period of 1970 - 1980** is known as the period of programming. The essential reason for starting to teach programming was not to perfect the programmers, but the conviction that it would develop students' logic and mathematical skills. At the same time, it was noted that some simple computer programs served as math exercises, even though the students could not deepen their mathematical knowledge for their trial-and-error character. These programs could simply serve to promote short-term memory. At this time in some developed countries in Europe, some programs for automated data processing were developed.

The late 1980s – early 1990s is the period of computer-based training (CBT) and multimedia with advanced graphics and sounds, with the pedagogical claim that students can achieve higher results in the learning process if it would be illustrated with animations and video clips.

This period is known as the flourishing period of CD-ROMs and multimedia and was expected to have a huge impact on the learning process in general and mathematics in particular. But in fact, what resulted was nothing more than a lukewarm result in deep and critical learning of mathematics. It was time for education policymakers to emphasize the strengthening of the interaction between mathematics, computer, and mathematics teaching. The importance of strengthening this interaction was directly related to the outdated methods used in teaching mathematics.

So in 1985, to study the situation and to draw attention to the impact of computers and informatics on mathematics teaching, the International Commission for Mathematical Instruction (ICMI) held a conference in Strasbourg, France in 1985, with the cooperation of UNESCO and concluded with a study precisely on the topic: *The Impact of Computers and Informatics on Mathematics and its Teaching* (Churchhouse, et al., 1985), in a Symposium held in Strasbourg, on1885

The study was undertaken for countries located in Europe and North America since the economic developments of these countries have allowed many schools and universities in these countries to be equipped with computers earlier than in other less developed countries.

The early 1990s. This stage is about introducing the World Wide Web. Using CD-ROMs had difficulty updating content on CD-ROMs. As the information being serviced changes very quickly, it was necessary to update it daily. This challenge would be solved by the Internet.

So we have the transition to internet-based computer training but without multimedia. Everything was limited to word processing, photography, simple animation, video, and audio editing. What was noted, was that the benefit, in the area of mathematics teaching was not very productive.

At this time, all concrete applications of geometry or calculus rely not so much on basic theories as on certain phenomena, in the same way, that computers and mathematical software work. In this way, students, who did not have good mathematical knowledge had a great benefit. In short, if mathematical software were used, students would accumulate more mathematical phenomena. Thus mathematics began to present comprehensible facts, but without mathematical explanations.

This way of learning is against the classical teaching methodology because the given way does not develop students' critical thinking, and ability to solve problems and apply mathematical knowledge in real-life contexts.

The late 1990sand early 2000s are known as the e-Learning period. Online learning in the late 1990s and early 2000s emerged in the form of e-learning. At this time we have a stream of requests coming from IT professionals across educational institutions toward education experts to build what was called the e-learning industry. In terms of mathematics learning, what was observed was the necessity of intercommunication between students themselves and between students and teachers, and this phenomenon was understood by the developers of different E-Learning platforms.

The end of the 2000s and the beginning of the 21st century. This decade is filled with innovations and discoveries (Table 1) that would have existed only in the most beautiful dreams of many, who devoted their entire lives to science. Undoubtedly, in parallel with this development of technology, came the intensification of the work of education policymakers to feel the benefits of this development in the field of education in general and in mathematics teaching in particular. In 2000, UNESCO first introduced the definition of ICTs and defined it as *the combination of informatics technology with other, related technologies, specifically communication technology.*

This definition implies that ICTs will be used, applied, and integrated into activities of working and learning based on conceptual understanding and methods of informatics (Khvilon & Patru, 2002).

Table 1. ICTs changed the world of education

Innovation		The role
	Apple iPhone	Completely transformed the world's idea of what a cellular phone is and what it can do.
	Windows XP	Very sophisticated operating system and at the same time a very efficient one.
	Google search engine	This highly functional search engine is the greatest achievement of this period.
	Social Networks	Provide an opportunity to share knowledge and experience.
	USB Flash Drive	Removable data storage device – compact and provides capacity.
	Wi-Fi connectivity	Internet access is almost everywhere or at anytime.

Understandably, this technological development would bring an immediate need to structure programs and take measures for technology-based learning.

This had nothing to do with programming, training, and practice, nor with multimedia and updates, but with a human world that shared thoughts and ideas. To reinforce this fact we would like to mention Mikhail Bakhtin and Lev Vygotsky who rightly hold that *all higher mental functions derive from actual relationships between human individuals* (Vigotski, 1930).

At the **beginning of the 21st century**, there is a discord between economic development, where people work in teams using a variety of technological tools, and on the other hand, education, which at first glance seems to have changed little from that of the mid-20th century even though supportive technology is developed. In most countries around the world, mathematics teaching follows the traditional way of explaining, where teachers are at the centre of reference in classrooms full of students and the role of technology is very limited.

It is for this reason that the Council of Europe met in Lisbon on 23 and 24 March 2005 (COUNCIL OF THE EUROPEAN UNION, 2005), urging the urgent need to adapt education and training systems to the needs of the economy, emphasizing the development of new basic skills such as technologies of information and communication. ICT was thought of as one of the key pillars of the strategy being undertaken.

Steps were taken to implement the e-Learning initiative, drawing up its action plan and program.

E-Learning was increasingly seen as the trend of the time in the academic world due to learning everywhere and at any time. To increase the efficiency of the use of e-Learning in teaching, it was necessary to use systems such as Intelligence Tutoring Systems (ITS) and Adaptive Hypermedia Systems(AHS) in such a way that, the first(-ITS) with an integrated expert (Phobun & Vicheanpanya, 2010).

The ITS monitor a learner's performance in math and personalize his lessons by adapting to his learning style and the Adaptive Hypermedia (AH), although less efficient offers how to create math skills. In 2010 an article in this issue describes a way to combine ITS and AH into an Adaptive Intelligent Tutoring System (AITS) for e-learning systems that allows knowledge to be stored in it.

They are not only independent from the field of knowledge but also support the preservation of knowledge transfer relationships. The conceptual results show that this innovative approach is useful for students in improving their learning achievements.

Technology and mathematics nowadays seem to have begun to find a common *language* to serve one another better. Both have changed in the presence of each other.

3.2 Benefits provided by ICTs tools in teaching mathematics

Recently in mathematics curricula, there has been a decrease in the number of hours that students have to practice with pencil and paper, overflowing with numbers and algorithms to remember. Increasingly, the place of endless actions has been occupied by logical structures of performing these actions and their applications in everyday life, while mathematical operations, as well as activities supporting the learning process, are being performed by technological means.

ICT tools provide students with computer simulations to illustrate solutions to problems that have a high degree of difficulty and are sometimes impossible to solve without technology.

Many mathematical activities, especially in the field of applied mathematics are strongly supported by technology. For example – in the analysis of data, where the computer allows students to use tables and graphs to organize, process, and then present and interpret the collected data.

Since 1997 (Crawford, 1997) defined these benefits concerning Information Technology (IT) in high schooldefines these benefits of Information Technology (IT) in secondary school:

1. IT is an interesting teacher. It can make learning easierand more attractive
2. IT is also a patient and responsive teacher. The software does not tired of waiting for a response. Computer-Aided Learning Software can give pupils immediate feedback
3. IT is pupil-centred.
4. IT supports open, independent, and flexible learning.
5. IT gives access to online learning resources
6. IT promotes sharing andcollaboration.
7. IT is blind to gender, race age, and disability.
8. IT is a valuable resource and unpredictable

Extraordinary attention is being paid all over the world to the mutual contribution that technology and mathematics make to each other.

More and more emphasis is being placed on the implementation of technology in mathematics teaching.

On the other hand, the strong foundations in mathematical education lay the foundations of computer and analytical thinking that further bring not only the development of technology but also the global education of today's children.

But let us list briefly, in general terms, the benefits that technology brings to mathematics education, ie what are some of the benefits of using ICT in general, and computers in particular in improving the teaching and learning of mathematics.

a. The **reinforcement of concepts through multimedia** plays an important role in sensory components and allows the use of a wide variety of learning modes.
b. **Visualizing of 2 and 3-dimensional geometric** figures through graphic illustrations encourages students to experiment with them. Gifted learners can visualize mathematical concepts in figures, and consequently through the use of technology they refine and extend this ability.
c. The ability of computers to enable us **to personalize learning** of mathematics. Many different software programs allow students to work according to their capacity and give them immediate and useful evaluation, and this has a very positive effect on student learning outcomes.
d. **Motivational and interactive computer skills**. It is a characteristic of pre-university students to want to deal with math exercises and problems that require a shorter duration to solve. They feel confident and motivated to try again when they have completed an exercise quickly and accurately. In classrooms where traditional teaching takes place this characteristic fades. Designed to offer students a simple first try, then another slightly trickier, and then another more sophisticated, different software programs keep students stuck in their world, without the fear that something went wrong during the problem solving and their teacher is going to see it. This fact removes them from the sense of feeling controlled by the teacher and makes them work more smoother. Collaboration between students and teachers and students themselves is evident by sharing ideas and projects via email and social media. Through computers, the quality of the projects and the work they carry out is presented through computer presentation, and with them increases self-esteem.
e. **Developing computational thinking through the use of ICT**. Computational thinking is seen as a necessity, not only for computer science but also outside the field. So in 1980, Seymour Papert introduced the concept of computational thinking in his book "Mindstorms: Children, Computers, and Powerful Ideas" (Papert, 1980).
   Papert regards computational thinking as a relation between different computer programs and thinking skills. The skills and competencies acquired through computer programs are the basis of computationalthinking in all fields in general and mathematics in particular. A different voice in this regard is that of Jeanette Wing, who brought computational thinking to another dimension, stressing that it should be an integral part of every child's analytical skills (Wing, Computational Thinking, 2006). Four years later, in 2010, Snyder, Wing, and Cuny, another group of scholars, brought it to a slightly more moderate form, where problem-solving was presented in a form applicable to even a computer program. (Cuny, Snyder, & Wing, 2010).

3.3 The most common uses of ICT and computers in general in teaching and learning mathematics

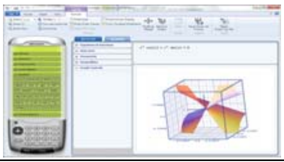
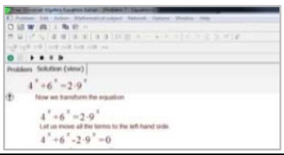
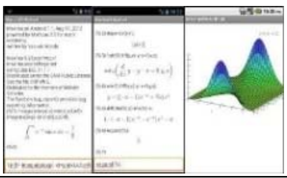
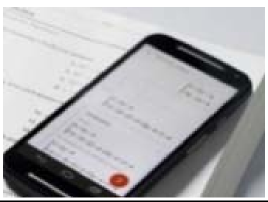
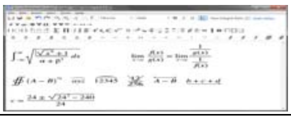
In the century we are living in, there is a flood of programs and software that not only facilitate mathematics teaching but also deepen students' knowledge in this field. In general, how the computer affects mathematics teaching can be classified into 3 major groups:

1. Tool software and software of teaching programs
2. Internet
3. Programming

In the mathematics of pre-university education, the first two are important.

The software offers help in a variety of mathematical topics (Table 2) ranging from algebra, and geometry to graphs, drawing different types of geometric shapes, and then proceeding to computation, statistics, and probability.

Table 2 The most common software in math education

Software		**Description**
	Microsoft Mathematics	Used for algebraic knowledge by helping students to draw 2D and 3D programs using a graphing calculator and to solve a complex equations by step-by-step argumentation
	Math editor	Serves to facilitate the work of students and teachers to write equations or polynomials, to store, modify and transport them to the documents
	Free Universal Algebra Equation Solver	Through this application, students receive sufficient help in solving and explaining equations, ranging from rational to exponential and logarithmic equations.
	Maxima	Used to solve various problems related to integration, and derivation, and works on all POSIX platforms such as Linux, Unix, OS X, and BSD. It uses Gnuplot for drawing.
	Photomath for Android	Solves equations by taking pictures
	TuxMath	Designed as a fun game, this app comes as an open-source program, helping children ages 7 - 13 learn math.

Software		Description
	Wolfram Mathematica	Excellent support for tasks and exercises of any mathematical type
	Pythagorea and Euclidea	Both Pythagorea and Euclidea are with a game-like design and include very challenging geometrical problems
	Euler Math Toolbox	A free tool providing anyone with a very efficient tool to solve most mathematical problems of almost all types.
	GeoGebra	A free math program that is available to both parties, teachers, and students
	Tibi's Mathematics Suite	Free software that helps teachers to easily explain geometry as well as accurately construct geometric figures and bodies.

To summarize any software, no matter how advanced or efficient it is, it cannot be told if it is good or bad. What can certainly be said is that it all depends on **how we use it and how much we use it.**

4 ICT DEVELOPMENT IN ALBANIA: POLICIES AND STRATEGY

4.1 An overview of ICT development in Albania.

At the time when programming had started in the developed countries of Europe, and even special programs were being developed that automatically processed the data, the availability of computers is also marked in Albania.

Thus, in 1971, the first big step was taken, and the first centre of mathematical calculation was established, which at that time was one of the departments of the State University of Tirana.

ICT use in Albania started in the eighties.

The first step of the introduction of technology in Albania was marked in 1971 with the establishment of the first Center for Mathematical Computing, which initially functioned as a department of the University of Tirana. The computers used were those of the second generation, and their focus was Applied Mathematics in various fields of human activity.

The second step was the creation of the Institute of Applied Informatics and Mathematics (INIMA) in 1985 as a project funded by the United Nations Development Program (UNDP). Thus was created the first metropolitan research and government network was. Unfortunately, this institute did not serve long, as the radical changes in the economic and political system in Albania in the early 90s, obscured the main role in the state policy and strategy that he had played until then, leaving it in oblivion.

In 2007 he was dismissed and replaced by a State agency under the auspices of the Council of Ministers.

In addition to INIMA, under the auspices of the Academy of Sciences in Tirana, digital technology and online systems were introduced at the Institute of Nuclear Physics.

In 1996, the online connection was realized in Albania through a UNDP satellite office in Tirana.

The substantial political changes that took place in the 1990s brought about a major change in the education system as well.

ICT departments were set up in many universities and a master plan was developed by the government for the use of ICT in teaching and internet access in all schools in the country (The Albanian Government, 1994)The government focused its work on **two main areas**:

1. Implement a cross-cutting strategy for information technology.
2. Develop a master plan for the school curriculum in the country.

The first aimed at rapidly developing and improving the ICT infrastructure and internet services throughout the country and **the second** aimed at equipping all schools in pre-university education in our country with computer labs and the Internet (The Albanian government, 2009)(Figure 3).

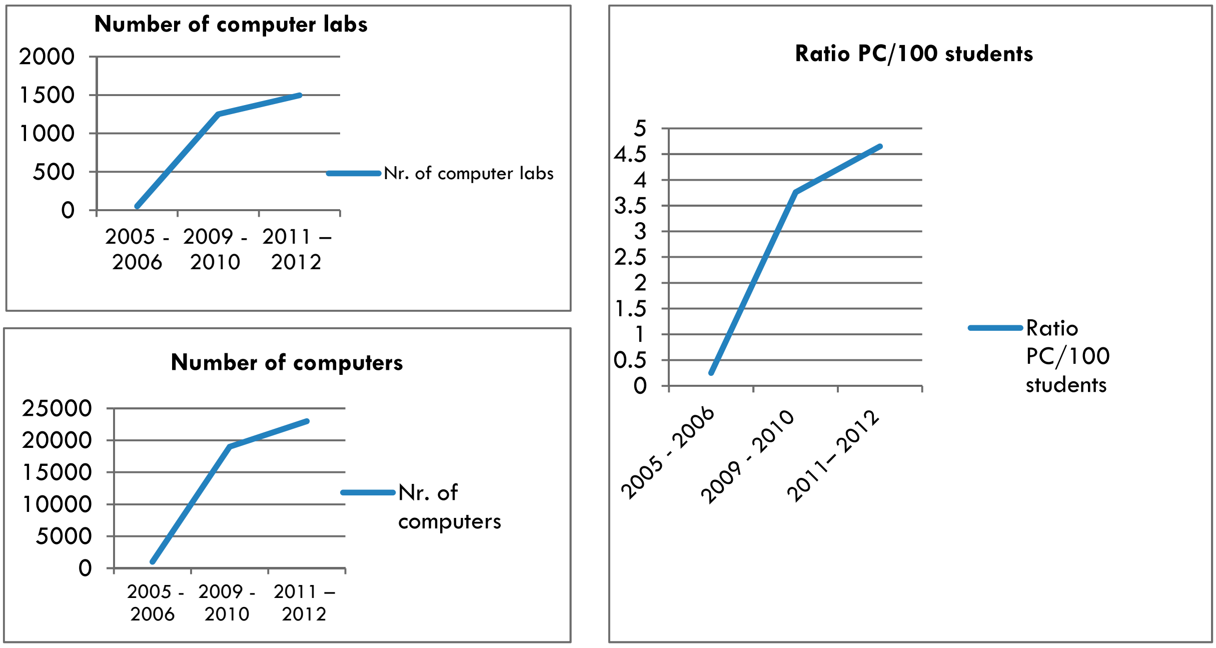


Figure 3 ICT infrastructure in Albania in al schools in pre university education

In 2006 ICT was studied only in high schools, while in 2014, ICT was introduced as a teaching subject from the 7th grade of lower secondary education.

An analysis conducted in 2014, (Shqipërinë, 2018) states that the Internet has been installed in schools to enable students and teachers to use different sources of information, as well as especially to assist in their work on curricular projects.Every school has a dedicated broadband connection, but this remains limited to computer labs.

Amongst the ICT issues in education, it is worth mentioning the following:

- The ratio of computer use per student varies from school to school; approximately, this ratio is 1:27 and in some cases even lower;
- Students may obtain information only in the computer laboratories, but not in the other school locations, i.e. the library;
- In more than 1/3 of schools, students have limited opportunities in obtaining information online;
- There is a complete lack of digital content in the mothertongue; only internet content that varies from the used source is used;
- There is a risk of exposure to inappropriate content;
- There is no awareness of children in internet utilization for the phenomenon, such as mocking, cyberbullying, or grooming.

Despite the investments already carried out, the utilization of ICT in educational institutions is limited. There is a need for more investments and attention to ICT in education to provide a society based on knowledge and to widely increase the skills of ICT utilization (MINISTRY OF INNOVATION AND PUBLIC ADMINISTRATION, 2018).

4.2 National policy and strategy for the development of ICT in education in Albania.

Although state policies regarding the implementation of ICT in education in Albania have always been and are ambitious, technological developments at an exponential pace always leave one step behind the use of educational technology in teaching.

Thus in August 2017, (UNESCO, 2017)it was stated that Albania has made significant progress toward expanding ICT access throughout the country, however, more can be done to expand ICT use and internet access in schools and communities in more remote areas.

The government is prioritizing the process of digitalization of education as an excellent opportunity to increase the quality of teaching, motivate students, and consequently improve their performance.

Specific objective A4, explicitly states the *Development of digital competence through better use of information and communication technology for teaching and learning*. (Albania. Ministry of Education and Sports, n.d.)

Concerning this objective in the strategy 2021-2026, the expected results, as well as the measures to be taken, are defined, as follows:

1. Creation of appropriate infrastructure for the use of ICT in schools and its maintenance, which contains:
    a. Setting standards for ICT equipment in schools.
    b. The installation of wireless networks in schools will be in function for the use of equipment by students and teachers.

c. Investment in adding digital equipment to schools.
d. Technical maintenance of ICT equipment through student clubs and local educational structure.
e. Development of ICT use policies at the school level.

It is understood that in parallel with the development of policies for the use of ICT in schools, the range of resources that enable ICT in teaching will be increased.

In (UNESCO, 2018), emphasis is placed on Open Education Resources (OER) which *have emerged as a concept with the potential to support educational transformation. While the educational value of OER lies in the idea of using resources as an integral method of curriculum communication in educational courses (ie resource-based learning), their transformative power lies in the ease with which these resources, when digitized, can be shared online.*

Such resources, which have extremely motivating values, enable mathematics learners, wherever students are, not only in their classrooms, but also in their homes, with their friends, and even while having fun. On the other hand, they are free and do not require any permission in advance.

But even though they exist, they are even free, to be used by anyone, students need to be oriented towards getting the best possible out of all these resources that exist on the internet.

For this reason in the National Education Strategy for the years 2021-2026, another important point was:

2. Development of digital competence through increased use of ICT in all subjects, which contains:
   a. Inclusion of digital competence in teacher standards.
   b. SELFIE piloting

SELFIE (Self-reflection on Effective Learning by Fostering the use of Innovative Educational Technologies), is an application designed to assist schools in incorporating digital technologies into teaching, learning, and assessment. This application is a good practice that is also guided by EU policies for lifelong learning (Council of European Union, 2018), which states *Learners, educational staff, and learning providers could be encouraged to use digital technologies to improve learning and support the development of digital competencies. For example, by participating in Union initiatives such as 'The EU Code Week'. The use of self-assessment tools, such as the SELFIE tool, could improve the digital capacity of education, training, and learning providers.*

c. Awareness of students about Internet security.
d. Setting standards for achieving digital competence according to the format of the European Union.

Involvementof lower and upper secondarystudents in the national coding program, through comprehensive commitments for their inclusion in specialized centres in support of the development of digital coding competencies for this age group.

Ministry of Education and Sports (MES) in Albania, in cooperation with the National Association of Independent Schools (NAIS), will develop standards for the achievement of digital competence in line with the "European Framework for Digital Competence", which defines five areas of competence and eight levels for their achievement. (Carretero & Punie, 2017)

At this point, it should be taken into account that teachers, in addition to being proficient in mathematics, must acquire the appropriate digital skills to carefully manage the subject programs offered by the curriculum, to integrate where and how well optimal technology is needed. to make it as productive as possible.

The quality of mathematics teaching in our country's schools will depend primarily on how often our math teachers will use ICT to reform new ways of teaching with a more student-centred focus on personalized learning.

If they have a clear vision to plan to teach based on the right technology, taking into account the needs and abilities of the students, they will also create active collaborative environments and personalize the teaching of mathematics.

Personalization of mathematics teaching requires that teachers have digital competence, which consists of several important components

Digital Competence of Educators (DigCompEdu) considers six different competency areas with a total of 22 competencies.

- Area 1 focuses on the professional environment;
- Area 2 on sourcing, creating, and sharing digital resources;
- Area 3 on managing and orchestrating the use of digital tools in teaching and learning;
- Area 4 on digital tools and strategies to enhance assessment;
- Area 5 on the use of digital tools to empower learners;
- Area 6 on facilitating learners' digital competence.

Areas 2 to 5 form the pedagogic core of the framework. They detail the competencies educators need to possess to foster effective, inclusive, and innovative learning strategies, using digital tools (European Commission, 2020).

Digital competencies mean, **first of all**, that teachers know the role, benefits, and opportunities that ICT tools will offer in teaching mathematics.

Secondly, they must be able to search, gather and select the right information that makes it possible to personalize the mathematics learner.In addition, they should be critical analysts of materials and platforms served on the internet, as not everything found on the internet can be productive for the classroom where they teach.

Third, after critical analysis of the platform or software chosen, they should reflect on it.

Undoubtedly, the coordination of all these components brings the immediate need to redefine the skills they currently have.

Future learning skills will include the ability to develop innovative ways of using technology to improve the learning environment and to encourage knowledge acquisition, knowledge deepening, and knowledge creation. (UNESCO, 2018)

In the UNESCO ICT Competency Framework for Teachers, three levels have been defined which show how a teacher acquires technology(Figure 4, Figure 5).

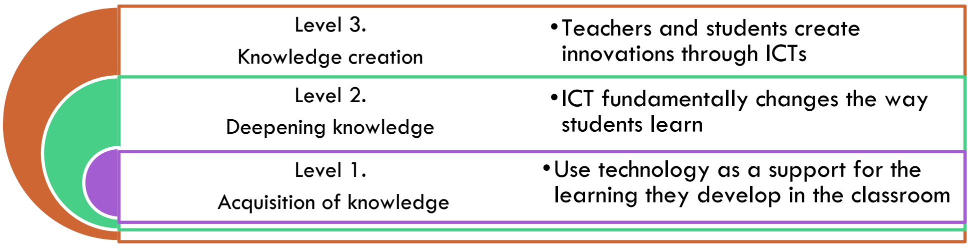


Figure 4Levels according to which the formation of digital competencies of the teacher passes.

These levels are combined by 6 aspects of their pedagogical work that go with a total of 18 digital competencies that a teacher must possess. In the document where the digital competencies of the teacher in ICT are defined, the table with 18 competencies that intertwines the pedagogical aspect with the digital one is given as follows in Figure 5

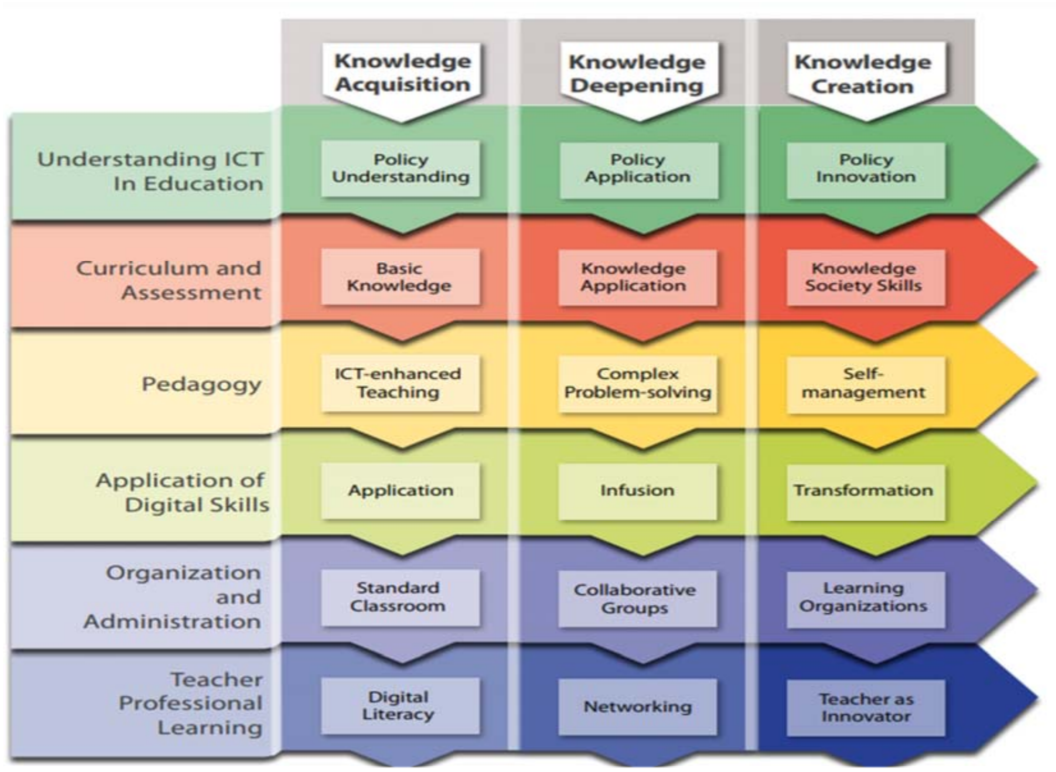


Figure 5The UNESCO ICT Competency Framework for Teachers. (UNESCO, 2018)

Each phase has its characteristics that require the right time and commitment to be fulfilled.They must be taken into account by education policymakers and seen as a priority to be included in the standards of teachers in Albania.

The challenge for math teachers is to make the gradual transition, of course, as quickly as possible, through all the cells of this table, reaching "Teacher as innovator". This achievement will undoubtedly bring Albanian society a development towards European standards and our children will be able to take maximum advantage of the boom of technological developments.

Finally, to form the digital competencies of our students and teachers, UNICEF Albania in cooperation with the Ministry of Education, Sports and Youth is supporting the initiative "Education Technology Centers". This centre will provide access to standard learning for teachers and students. UNICEF has distributed 840 tablets and 48 smart tablets, tablet charging and storage devices, and free internet to 24 compulsory schools (Unicef, 2021)

The fact that these devices are available and given access to teachers and students according to a set schedule and program, makes it possible for teachers to plan technology-supported lessons.

In parallel with the assistance of ICT tools, teachers will be trained to use them effectively, orienting learners towards active teaching methodologies, of which Science, Technology, Engineering, Art, and Mathematics (STEM) will be an asset added to this initiative undertaken by UNICEF.

In the Lifelong Learning Competencies Development (LLCD) material, (European Union, 2018) which addresses the challenges of lifelong learning-oriented competencies, Science, Technology, Engineering, Art, and Mathematics(STEAM), is identified as one of the best practices for overcoming these challenges.

Learning methodologies such as inquiry-based, project-based, blended, arts- and games-based learning can increase learning motivation and engagement. Equally, experimental learning, work-based learning, and scientific methods in science, technology, engineering, and mathematics (STEM) can foster the development of a range of competencies (European Union, 2018).Today's educational policies in Albania have begun to transcend traditional didactics by adapting teaching to new technological conditions.

The new trend in teaching, that of blended learning, which has started to be applied in 24 schools in Albania is based on some important principles which not only enable students to communicate interactively with the information they will learn as well as together but it trains teachers to personalize teaching, adapting the technology curriculum to supporting technology curricula.

The integration of digital materials with the traditional approach of teaching, makes blended learning, one more opportunity to use new teaching methods, such as distance learning, gamification, etc.

Important principles that underlie it, such personalizing the learning, supporting progress according to skills and needs, increasing student motivation, increasing learning resources, use of easily accessible applications by anyone it is a learning methodology that personalizes the math learner.

This initiative taken by the Ministry of Education in Albania and which has started its implementation by training 350 teachers throughout Albania, is progress towards strengthening the role of ICT in teaching. In one survey, based on performance indicators from the International Society for Technology in Education (ISTE), two scales were developed that measured teachers' self-perceived ability and their frequency of usage of technology integrations made with 179 teachers in Taiwan and showed a positive correlation between teachers' technology integration ability and usage (Hsu, 2010).

In the extended UNESCO report, in the Education Policy Analysis: Issues and Recommendations for Albania, in 2017, it was recommended to use ICTs in education as well as to increase the internet connection, especially in rural areas, to invest in its digital resources, learning and prepare national guidelines on the use of ICT in education (UNESCO, 2017).

4.3 Overview of competency-based learning in mathematics in Albania.

The quality of teaching in general, and that of mathematics in particular has been at the constant attention of all educational policies in Albania.

In 2014, the Ministry of Education, Sports, and Youth (MoESaY) of the Republic of Albania reformed the legal framework of the Curriculum Framework. (The Curriculum Framework is a key written document, which provides essential guidelines and instructions in the design and implementation of the Albanian pre-university education curriculum).

Many reforming elements were defined in the curricular framework of the Republic of Albania.**Competence-based learning** and **student-centred learning** are the 2 most important reforming elements of this interrelated curricular framework because both focus on students and the process of learning.

The competency-based curriculum shifts the focus from subject-centred mathematical learning to student-centred learning situations and active teaching.

Student-centred teaching offers different learning situations through which the student is formed in the social, cultural, intellectual, and civic aspects. The key competencies defined in the mathematics curricula include the integration of skills, abilities, knowledge, attitudes, and values to achieve active citizenship, social inclusion, and sustainable personal development.

The design of competency-based curricula is in line with the European Recommendations for lifelong learning, which defines mathematical competence as the *ability to develop and apply mathematical thinking to solve a range of problems in everyday situations. Building on a sound mastery of numeracy, the emphasis is on process and activity, as well as knowledge. Mathematical competence involves, to different degrees, the ability and willingness to use mathematical modes of thought (logical and spatial thinking) and presentation (formulas, models, constructs, graphs, and charts)*. (European Parliament and the Council, 2006).

Competency-based teaching is based on Bloom's 6 levels of the taxonomy (*remember, understand, apply, analyze, evaluate and create*). (Bloom B. S., 1956)

All these levels have access to the 6 competencies that students must possess at the end of each subject program of mathematics at the end of each school year.

Briefly, they could be summarized as follows:

1. **The solution to the problem situation** includes the same time:
    a. Description and solution of problem situations on a practical level, taken from the common experiences of daily life.
    b. Describing and solving abstract problem situations by developing intellectual capacity and creative intuition.

This does not mean that problem-solving with context from real life mechanically sets the mathematical background in the function of problem-solving, but from a situation with context outside mathematics, it helps to mathematize the problem situation, developing the skill for mathematical modelling at the same time.

2. **Mathematical reasoning and verification**, which includes:
    a. Using reasoning, argumentation, and proof as basic aspects of mathematics.
    b. Logically organizing facts, ideas, or concepts to achieve a more reliable result than intuition.

It is no coincidence that the main weight of this competence is reasoning.

Reasoning as a concept is very broad and contains in itself the deductive, inductive, and culminates with the hypothetical-deductive (Piaget & Cook, 1952), about which Piaget says that it is the last stage of development.

If we were to include logic here as a form of reasoning, which occupies a primary place in mathematical thought, the panorama of all the components of reasoning that occupy a place in the mathematics curriculum would be completed.

Through mathematical verification and logical organization of facts, these functions defined by Gila Hanna (Hanna, 2000) are realized simultaneously in her article "Proof, explanation, and exploration: an overview":

- verification (concerned with the truth of a statement)
- explanation (providing insight into why it is true)
- systematization (the organization of various results into a deductive system ofaxioms, major concepts, and theorems)
- discovery (the discovery or invention of new results)
- communication (the transmission of mathematical knowledge)
- construction of an empirical theory
- exploration of the meaning of a definition or the consequences of an assumption
- incorporation of a well-known fact into a new framework and thus viewing it from a fresh perspective. (Hanna, 2000)

Mathematical reasoning has always been the focus of PISA-s draft directives, so even in the PISA 2022.

It is clearly stated that: *Mathematics is a science about well-defined objects and notions that can be analyzed and transformed in different ways using "mathematical reasoning" to obtain certain and timeless conclusions. In mathematics, students learn that, with proper reasoning and assumptions, they can arrive at results that they can fully trust to be true in a wide variety of real-life contexts. The goal is to reason mathematically and not to reproduce routine mathematical models.* (OECD, 2018)

3. **Mathematical thinking and communication**

This includes communication through reading, writing, discussion, listening, asking to organize and clarify mathematical thinking, as well as clear and logical communication between teacher and student and between students with each other.

On the other hand, this competence aims for students to analyze and evaluate mathematics as well as the thinking and strategies used by others, always using correct mathematical language to express themselves. It is the task of teachers to stimulate these skills through personalized work and attractive and innovative learning activities.

4. **Conceptual connection**, which includes:

a. constructing mathematical concepts to form a whole using the dependencies between these concepts.
b. developing connections between concepts, through mathematical reasoning, constructing and applying them to relevant mathematical processes.

This competence aims for students to see mathematics as a chain of knowledge where everything is in the function of everything, and all together are in function of creating competent citizens to apply mathematical knowledge in real life.

5. **Mathematical modelling**, which includes:
    a. Describing and creating models using basic mathematical operations in everyday life situations.
    b. The process of presenting the situation from real life with mathematical language.
    c. Finding the mathematical solution through the use of relevant techniques, which is then interpreted in real life.

Mathematical modelling is a cyclical process in which real-life problems are translated into mathematical language, solved within a symbolic system testing solutions again in real life. (Verschaffel, Greer, & de Corte, 2002)

All the process described by the authors above goes through these 4 stages: (Capone, 2022)

- *Identification of relevant mathematical tools and reorganization of the problem based on mathematically identified concepts;*
- *Elimination of elements of reality from the problem;*
- *Solving the mathematical problem;*
- *Interpretation of the mathematical solution in terms of the real situation.*

6. **The use of technology in mathematics**, which includes:
    a. The use of technology as a tool to solve or verify solutions.
    b. The use of technology as a means of gathering, communicating and discovering information.

In all work processes, technology has more and more great use. Recalling the objectives of PISA 2022 can quote: The technology change is also creating the need for students to understand those concepts of computational thinking that are part of mathematical literacy.

Using educational technology in our classrooms while teaching mathematics, educate students who will not only use deductive or inductive mathematical reasoning but will prepare the students to perfect mathematical thinking through computational, logical, and critical thinking.

Although at a slow pace, the reforms made to the new curriculum have begun to bear fruit in the math tests developed by the International Student Assessment Program (PISA).

Table 3, presents the results of Albanian students obtained from 2000 to 2018 in mathematics, in the test that the Organization for Economic Development and Cooperation (OECD) developed through the International Student Assessment Program (PISA) (OECD, 2018).

Table 3 The results of Albanian students, obtained from 2000 to 2018 in mathematics in PISA

Year	2000	2009	2012	2015	2018
Points	381	377(-4)	394(+17)	413(+19)	437(+24)

In 2018, about 58% of students reached level 2 or higher in mathematics, compared to the OECD average which was 76%.

This category includes students who can interpret and know, without direct instruction, how a simple situation can be presented mathematically. On the other hand, only 2% managed to have a level of 5 or higher 5.

This percentage remains very low compared to the 11% level which is the OECD average. This category is for students who can model complex situations mathematically.GraphicFigure 6 presents this trend.

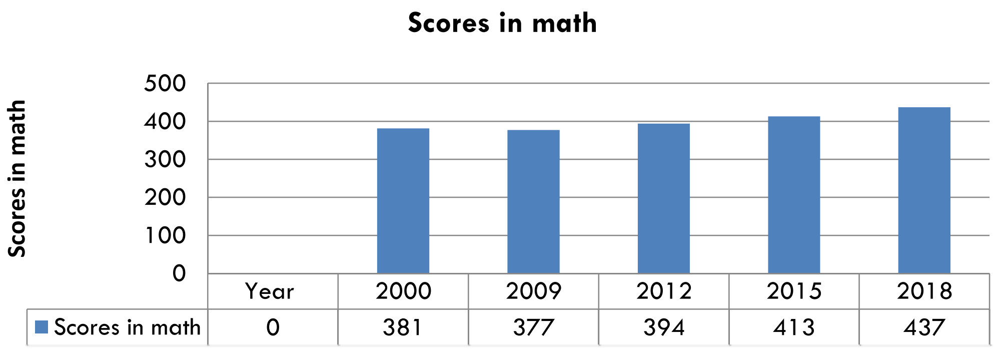


Figure 6 Scores in math (PISA, Albania) from 2000-2018

The educational policies and the regulatory framework for education in general, and that of mathematics in particularin Albania, are in the right direction, but the results are coming at a slow pace.

5 Conclusions

ICT tools supporting education asa whole, but especially in mathematics in particular. Technological education tools are playing an important role in modernizing mathematics learning and shifting it from teacher-centred learning to student-centred learning as well as competency-based learning. Shifting the focus to active teaching/learning requires ICT tools support.

Some of the benefits it offers in the field of mathematics teaching/learning are:

1. As ICT tools facilitate and accelerate the performance of operations, mathematical curricula are oriented towards the application of logical structures for performing operations and their application in everyday life, by delegating routine mathematical operations to ICT tools.
2. Reinforces concepts through multimedia by playing an important role in sensory components and allowing allows the use of a wide variety of learning modalities.

3. Develops abstract thinking, strongly supporting problem-solving strategies, project-based learning, question-based learning, etc.
4. Provides computer simulations as illustrations for solving problems that present a high degree of difficulty.
5. Visualizes the learning process while increasing the level of understanding and facilitating the transition from concrete to abstract thinking.
6. The variety of programs, software, and computer platforms it offers enables students to learn according to their steps.
7. Increases the motivation of students to deal with mathematics.
8. Develops computational and analytical thinking, as the skills and competencies acquired through computer programs are the basis of computational thinking in all areas in general and mathematics in particular.

RECOMMENDATIONS

Regarding the current context in Albania.

It is a fact that state policies have been and continue to be very ambitious regarding the implementation of ICT in teaching. This is reflected both in the curriculum frameworks of the mathematics curricula and in the National Strategies for the Development of Education in general.

These ambitious policies are focused on:

Creating the appropriate infrastructure which starts with setting standards for ICT equipment in schools, investing in digital equipment, and their technical maintenance, the development of policies at the school level.

But, on the other hand, it is a fact that the current situation in Albania needs intervention in this direction.

This intervention should be done in three main directions in parallel:

1. Teachers are continuously trained in two directions, firstly to personalize teaching based on active methodologies, and secondly for meaningful use of ICT in schools, not only as a facilitator of their daily work at school but as a supporter of the learning process as a whole, supporting students to advance according to their abilities and needs in the learning process.
2. Creating an European model of teachers with the characteristics set out in "The UNESCO ICT Competency Framework for Teachers" by setting up a structured system of policies and priorities, based on an analysis of the strengths and weaknesses of our teachers in terms of competence digital.
3. Taking measures to acquire digital competence in terms of teaching mathematics at the university, coordinating work with universities that prepare teachers to introduce as a special curriculum the preparation of technological didactic tools for teaching.

Although measures have been taken one after the other and successive reforms have been carried out to bring the model of the Albanian teacher closer to the model, defined by UNESCO, as long as they are not giving the right results, they show that they have either been partial or not are properly implemented.

The results In PISA from a year to the next one speak of a very modest change that mathematics education has undergone in Albania.

Two of the fundamental changes that have been made in the curriculum of mathematics education in Albania, competence learning and student-centred learning require first of all the strengthening of all the potentials that our students possess, starting from the emotional to the intellectual onesin the field of mathematics. For this empowerment to take place there is an immediate need to personalize mathematics learning through ICT.

The challenge of learning mathematics in the 21st century around the world is personalized learning, therefore it remains a challenge for the education of mathematics in Albania.

CHAPTER 2. PERSONALIZED LEARNING IN MATHEMATICS

Every child has a different learning style and pace. Each child is unique, not only capable of learning but also capable of succeeding

Robert John Meehan

In what ways do researchers define personalized learning, and what is its meaning in mathematics?

What are the components of personalized learning and what are their characteristics?

What is the role that personalized learning plays in the mathematical form of students in lower secondary education?

What tasks does the math teacher fulfil while personalizing the learners?

What are the strong influences that personalized learning through ICTs brings to the formation of students as responsible citizens?

1 PERSONALIZED LEARNING COMPONENTS

Personalizing mathematics teaching is a challenge for today's mathematics teachers, not only in Albania but all over the world.

Ambitious teachers all over the world are perfecting their day-to-day work towards personalized learning, with the support of ICT tools, especially after the COVID-19 pandemic experience.

First of all, let's bring some different definitions of personalized learning.

*The termpersonalized learning, or**personalization**, refersto a diverse variety of educational programs,learning experiences, instructional approaches, and academic-support strategiesthat are intended to address the distinct learning needs, interests, aspirations, or cultural backgrounds of individual students.* (Great Schools Partnership, 2014)

The same source it is presented also another definition of personalized learning seen from a slightly different perspective;

*Personalized learning may also be called**student-centred learning**, since the general goal is to make individual learning needs the primary consideration in important educational and instructional decisions, rather than what might be preferred, more convenient, or logistically easier for teachers and schools.* (Great Schools Partnership, 2014)

Formulated in slightly more concise language, comes the following definition in which it is clear that the essence remains almost the same as the first two.

Tailoring learning for each student's strengths, needs, and interests–including enabling students' voice and choice in what, how, when, and where they learn–to provide flexibility and support to ensure mastery of the highest standards possible. (Slocum, 2016)

In 2017, The National Education Technology Plan (NETP) of the United States 2017 has defined personalized learning as follows:

***Personalized learning** refers to instruction in which the pace of learning and the instructional approach are optimized for the needs of each learner.Learning objectives,instructional approaches, and* *instructional content (and its sequencing) may all vary based on learner needs. In addition, learning activities are meaningful and relevant to learners, driven by their interests,and often self-initiated.* (Department of Education, U.S., 2017)

But what **does it mean to personalize teaching and learning in math**?

To personalize the teaching and learning of mathematics means putting students at the centre by orienting them to build all learning processes according to their needs, abilities, and talents and through the processes of education good mathematical reasoning, as well as logical, critical, abstract, and computational thinking.

Personalizing math learning also means educating creative citizens who are flexible about learning different types of.

Personalized learning requires a job that is both qualitative and voluminous on the part of mathematics teachers, as it requires organizing the whole learning process based on learning with competencies as well as on students' learning styles and student centralization in the process. *Educators who are more successful in personalizing learning embrace the belief that: Students are more than content. We need to treat students as unique individuals with strong points, interests, and perspectives.* (Education Elements, 2016)

If the whole process of personalizing learning is done without the help of educational technology, this would be a colossal job on the part of math teachers, if we consider that it would even be possible.

All this for the fact that the teachers are not the "tap" that fills the minds of students with knowledge but are the catalysts and coordinators of all intellectual processes that have to do with the growth of students as individuals in the first place and then with the formation of skills and competencies in mathematics that they must master. Teachers in general, but especially those in mathematics, should be promoters of students' intellectual growth, as well aslogical and critical reasoning, aiming at their global education.

The basis of personalization of the whole process of mathematical learning begins with the students, their needs, and abilities being identified to make their aspirations and goals a part of the process, as well as their needs, skills, talents, and mathematical content that they will acquire. *The whole process ultimately aims to educate self-oriented and self-confident students who move forward forming skills and competencies to reflect, improve, and perfect. Despite the uniqueness of each student, there are some decisive processes where personalized learning strongly influences.* (Education Elements, 2016)

These processes are:

1. Building awareness of their uniqueness, strengths, interests, and learning modalities
2. Empowering them to protect themselves and their community
3. Developing self-management skills that encourage their personal and academic growth
4. Inspiring students to remain students throughout life.

All the above characteristics ultimately emphasize the final focus of education as a whole, which is the formation of global citizens, capable of solving all the challenges of life with maturity and rationality.

In the same article, the 4 components of successful personalized learning are defined.

1. Reflecting on students and setting their goals;
2. Targeted instructions;
3. Flexible paths and pace;
4. Collaboration and creativity;

Bellow, an analysis of these 4 components with some simple examples is provided to identify the characteristics of these components in the mathematical context.

1.1 Reflecting on students and setting their goals

The teacher, while working to achieve the short-term goal of the lesson, aims together with the students to develop mathematical skills and competencies and to set priorities for intermediate and long-term learning. The expected results should always be specific and aligned with the intermediate and long-term goals so that the student can see a clear way forward.

Making students reflect on what they have learned or on a mathematical concept that they will need to use in future lessons is a process that requires first and foremost coordination between the mathematical concept they are learning and the application of that concept. In short, they always want to know the purpose of learning a concept.

Naturally, the goal can be evidenced by a learning situation that can be carefully chosen and which can push students to construct the concept as well as to use it further. The teacher and the student in collaboration should set up standards that allow the student not only to touch the predicted result but to look far, into the connection of the concept with other knowledge that will follow until its implementation in real life.

The cases when the teacher brings a situation from the real context and tries to build its mathematical model together with the students are cases that enable a greater reflection of the students regarding the pure mathematical concept that they are learning.

Let's concretize it with a simple example observed while teaching the concept of “Least Common Multiple” (LCM) of two or more numbers together.

The teacher could present to the students this simple situation of learning: "One flower is watered once in 3 days and another once in 2 days. If today, both flowers are watered, after how many days will both flowers be watered again at the same time?

Students will learn the same concept even if they are simply asked to find L.C.M. of numbers(2; 3), but wearing the concept with "life" makes math more beautiful and engaging, and answers the main student'squestion: *Why do I need all this mathematical knowledge*?

Let's dwell again in the same situation.

The concretization of the situation or the **construction of schemes for understanding the essence** plays a very important role.

The construction of the schemes is well **accompanied by the pedagogical simulation apparatus,** which stimulates their thinking toward the achievement of the final goal.

To solve the problem with three simple tools, two loops: a circle, and a square, as well as a matchstick, students are encouraged to create the mathematical model as in the picture (Figure 7),

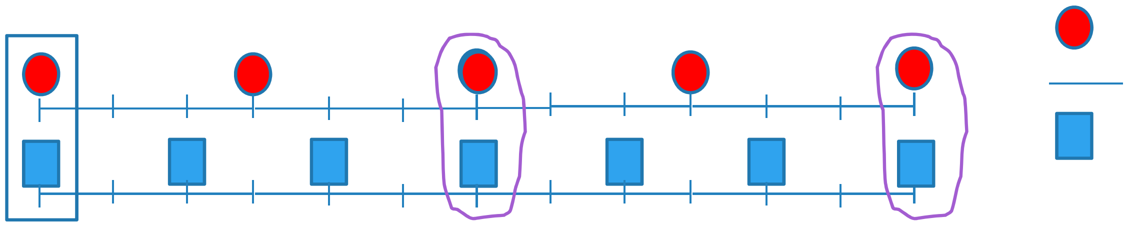


Figure 7 Graphic illustration of situational solution.

The pedagogical simulation apparatus is built through tasks and questions such as:

- *Mark with numbers 6 times from the watering of the first flower. (3, 6, 9, 12, 15, 18….)*
- *Mark with numbers the 6 times from the watering of the second flower (2, 4, 6, 8, 10, 12….)*
- *After how many days will they be watered again at once? (6, 12, 18, …)*
- *When are they watered for the first time at once?*
- *On what day are the flowers watered for the first time at the same time?*
- *How many times will this be for the first flower?*
- *What about the second flower?*

The students investigate, why the shortest time limit is 6 days, and the flowers are watered for the third and fourth time according to the scheme in the picture.

In the first steps, the students can write the multiples of the 2nd, then 3rd, and after identifying the commonalities they find the smallest common multiple of both, and then understand the mathematical sense of the solution of the problem. This moment is very important because it presents to them the connection that mathematics has with real life. Meanwhile, they also learn the purpose of why they learn how to find the L.C.M. of two numbers, where it can be used in everyday life. The teacher develops the competence of mathematical thinking by enabling the use of this concept further in real-life situations.

This mathematical concept, treated in a lesson, could be extended by a preliminary experiment in the form of a practical task:

- *Choose two different flowers in your home. Water them as a start on the same day. Water one every 3 days and the other every 2 days. Keep notes of how many days will you water the flowers again at the same time.*
- *Predict how many times you will have watered them at the same time when 18 days have passed.*

The effort to distinguish the students' levels expands the range of requirements that make the student deepen in knowledge and connect the concept with other mathematical knowledge.

The way the math teachers put goals for their students, and how they organize the whole learning process, based on the inclusion of all students' senses, guarantees the continuity of the whole process of personalization, passed from concrete thinking to abstract. Each student must take his or her own time for reflection depending on his / her Personal Profile Learning (PPL).

If the student takes the space and time to learn while increasing the range of examples from reality, during the process of thinking according to his/her profile, the self-esteem, and confidence that he/she can reflect in the final goal increases.

1.2 Targeted instructions

This component of personalized learning begins with an initial study of the student's family background, the intellectual formation of his or her parents, and stories from the informal or informal mathematical formation that the student may have.

Based on these initial data, as well as the academic level that the student has in mathematics at the time the plan is drafted, and the way his progress has evolved, objectives are set for that student.

Also, a very important component to consider is his abilities, strengths, and weaknesses, along with his preferences and learning styles.

Finally, a list is made of the factors that drive learning and sets out the techniques, methods, and strategies to be used to achieve it.

Everything mentioned above is summarized in a PPL for each student.

PPL is an individual document for each of the students of a certain class that is built from the beginning of the learning personalization process and that can be modified and changed throughout the year.

Students' mathematical thinking and the formation of their mathematical competencies are processes that are subject to constant change and evolution.

This is a very important fact to keep in mind by all math teachers as they personalize their learning.

It should be used consistently and should be referred to, to find the teaching strategy or method to be used, based on their learning style, needs, and abilities. On the other hand, referring to the PPL, the teacher has a clear picture of the type of technology that will be used under the needs of the skills that the student has so that the process of forming his mathematical competencies is optimal.

PPL is built based on four frames:

1. Demographics
2. Academics
3. Learning capacity
4. Aspirations learning drivers (Digital Promise)

A suitable model, based on Digital promise (Digital Promise)would be a student form as in Table 4.

Table 4 Student Personal Profile Learning Form (PPL)

Name:…………..	**…………………**	**Subject:**		**Class …….**
Frames	**Characteristics**			
	General Identification	Family/living arrangements	Family history of formal learning	Other information

Demographics				
Academics	Test scores	Progress data	Formative/interim assessments	Current academic goal(s)
Learning capacity	Skills	Habits	Dispositions	Current non-academic goal(s)………
Aspirations, learning Drivers	Current preoccupations	Hopes for the future	Factors that propel learning	Other goals

Referring to PPL, the teacher builds the structure of the student groups and plans specific instructions for each group in the function of the strengths or weaknesses that each of them has.

Instructions are also planned based on their learning style, their preferences for learners, methods, and techniques that have previously been proven to be productive. At this stage of the process, the student participates indirectly in the selection of guidelines for mastering the subject competencies, as it is his / her learning profile that dictates this selection.

On the other hand, he can directly suggest activities that contribute to the improvement of the learning process, activities in groups of students to collaborate on projects or practical tasks.

It is very important that **at this stage nothing is left to chance**, even the constant changes in the student's learning profile should be seen as a priority to choose the right methods and techniques at all times.

An important aspect of this process is also **finding supportive educational technology**, to make learning as attractive and efficient as possible.

The teacher needs to identify students' learning styles in advance and, depending on the learning style, research the platform, software, or technological tool they will use.

Depending on the barriers he may have in his classroom regarding technical support, he can choose another alternative where it is known that students have access, such as the cell phones they have, that students can play and learn at the same time.

Strategies for personalized learning are very complex,because the adaptation of the goal, with the way to achieve it, must be done together with the student. The goal is, as the student learns mathematics, to reason critically and logically, and progress from the concrete to the abstract. In parallel with this progress, he reinforces computational thinking, and gradually constructs the competence of independent learning.

This competence is very important, as it is the basis for educating rational and disciplined citizens, who have a perfect work education, and who can plan strategies and guidelines and provide solutions to the problems of everyday life.

1.3 Flexible path and pace

A look at the example, presented at the beginning, reveals that the teacher should offer different opportunities to reach the final result, and even the proposed possibilities may be such that everyone learns at his own pace. An *open door* for flexibility remains technology, which with the variety of mathematics education software it offers, enables a wide range of students with different learning styles to get the most out of it.

Everything it makes available is tailored to their abilities and needs.

The opportunities for benefit must be provided directly by the teacher, they should be diverse and include all student profiles. It is up to them to choose based on their learning style or styles and their rhythm. They can not remain indifferent, there is already an infinity of platforms and technological tools in circulation that support the mathematics teacher and that can offer to each student, according to his/her level, a wide range of supporting knowledge. They are there, not to replace the role of the teacher, nor to be the "learning superpower" that brings the desired results, much less to be the only tool that brings about a great positive change in the direction of learning.

1.4 Collaboration and creativity

This component of personalized math learning should be kept in mind by math teachers, as *the 21st-century learning skills are often called the 4 C's: critical thinking, creative thinking, communicating, and collaborating. These skills help students learn, and so they are vital to success in school and beyond.* (UNIVERSITY of NEBRASKA–LINCOLN, 2016)

Being creative requires dedication, ideas, knowledge, and above all passion. Depending on the purpose of the lesson, the lesson may begin with a brainstorming session or a learning situation that arouses interest in the classroom and increases curiosity about the final solution. Improvising a didactic tool to absorb the mathematical concept, why not even a premeditated didactic tool, can arouse students' interest in learning mathematics. The important thing is not to be easily predictable by students about what you will bring to a class.

Classes should not be a copy of each other in terms of their routine activities.

Creativity in the lesson should be comprehensive, it should start from small didactic tools or improvised examples at the moment, and go to the imagination of ideas for applying the mathematical concept in practice. Inclusive creativity also means cooperation with students and their active involvement in classroom activities.

Giving responsibility to each according to their abilities and encouraging them to achieve the purpose of the task given to them while being cooperative and oriented toward the final goal, makes students responsible for the learning process and at the same time strengthens their decision-making.

A lesson based on creativity and collaboration can also begin with the individual work of students which is strictly based on instructions given by the teacher, continues further work in pairs or small groups, being interested in discussions and giving ideas, sharing roles for their work, and taking responsibilities in performing a task or solving a problem. Their collaboration and creativity are very evident especially when active teaching methods are applied such as project-based learning, problem-based learning, or question-based learning.

All of these active methodologies have at their core **collaboration** and **creativity**.

The implementation and finalization of such learning processes encourage students to think independently and have the autonomy to make decisions about problems that are sometimes much more complex than those provided in the curriculum frameworks of the mathematics curriculum.

And, while the math teacher is working to personalize the math teaching, he is simultaneouslyperforming the following important tasks (Table 5).

Table 5 Range of tasks to be completed by the math teacher as he/she personalizes the teaching and learning of mathematics

Teacher	Important tasks to be completed by the math teacher to personalize learning
Recognizes	• The needs and interests of students • Their strengths and weaknesses • Their multidimensional skills (starting from personal ones to those in the field of technology) • How long does it take for everyone to master each of the mathematical concepts? • Gaps they have concerning previously learned knowledge • Learning modalities as well as their learning profile
Selects	• Appropriate active teaching and learning strategies • Technological platforms and materials that will be implemented during the active teaching • Topics that will be intertwined, while conceiving learning on a project basis or in group work • The model they will design to implement in teaching • Techniques and tools with which to intervene in students' weaknesses to improve them, while working hard to strengthen their strengths. • Combinations of personal skills and those in the field of technology will combine to realize the process of teaching mathematics
Prevents	• The situation of despair in the face of difficult concepts • The situation of pessimism associated with the thought "I am not made for learning mathematics" • Abandoning the desire to do the math. • Getting away from the core while you are learning math. • Deepening the weak points in teaching mathematics. • Superficial thinking while doing math.

Teacher	Important tasks to be completed by the math teacher to personalize learning
Guides	• At the right time • In the right place • At the right level • According to the need and abilities of each (to personalize mathematics learners) • The path to critical, logical, and analytical thinking. • The platform or technological tool to be used.
Encourages	• Independent thinking to structure meaningful tasks • Creative thinking further develops social skills as well as what they already know from experience. • Logical thinking is to organize their thinking while trying to successfully realize the product of their work. • Building in-depth knowledge of mathematical content, thus developing critical thinking. • The spirit of cooperation and the desire to work in a group. • The desire to learn math.
Creates	• Competent citizens in the field of mathematics and beyond. • Collaborating citizens, who are creative and know how to collaborate to build together skills and competencies. • Global citizens who know and value their values. • Capable citizens from the decision-making, and managerial point of view. • Citizens who know how to work in a team, manage time, synthesize information, and use technological tools. • Students and managers of the mathematics learning process are being led and mentored by him.

Throughout the process described above, it is clear that the professional competencies, including the scientific and psycho-pedagogical ones possessed by teachers, must be combined with their very good digital skills.

At each stage of the process, the educational technology simultaneously supports not only the organizational part of the process (eg., electronic files for the student's profiles (PPLs) which contain the table of characteristics of each student, the individual worksheets for each, the plans of the students. detailed follow-up of each according to skills, different tests for certain periods, etc.), but the whole process, offering help and support with math software, math games, and various applications that create a comfort zone for students, and why not also pleasure.

Since 2006, Sanna Järvelä (Järvelä, 2006)have been clearly defined 7 critical dimensions of personalized learning that are:

- The development of key skills which are often domain-specific.
- Levelling the educational playing field through the direct improvement of students' learning skills.
- Encouragement of learning through building motivation.

- Collaborative knowledge-building – new learning environments in education and the workplace are often based on shared expertise, based on three elements: progressive inquiry, problem-based learning, and project-based learning.
- New models of assessment on which personalized learning is seen to depend, such as authentic assessment, direct assessment of performance, and digital portfolios.
- Use of technology as a personal cognitive and social tool.
- Teachers are key: new learning environments require complex instructional designs and teachers will need to be strong in communication and collaboration.

Students are more motivated to learn math if the variety of tools and methods chosen to use in math learning fits the way they spend their free time. And their free time is dominated by the access they have to ICT tools.

So, they are motivated and expect that ICT tools will be included in all phases of learning, starting from the way the lesson is organized, their involvement in it, and the different ways of assessment. They are more motivated if they receive a real-time assessment of a task they are completing and each aspect of learning.

The goal of today's teachers is to turn traditional classrooms into classrooms that combine different types of student-centred active teaching methods and techniques, with clear ideas of what to achieve with each student and what technology they will use to achieve this goal.

Technology plays an important role in the differentiation of learning, in the guidance given, and in the assessment of students. *A pedagogical shift in technology engenders new teaching approaches to expand students 'conceptual understanding, procedural fluency, and strategic competence in mathematics. ICT makes mathematics teaching healthier and helps to increase the achievement of students*. (Gera & Verma, 2012).

2 IMPACTS OF PERSONALIZED LEARNING THROUGH ICTS TOOLS ON MATHEMATICS EDUCATION

Personalized learning in general and mathematics in particular is a very delicate and very complex problem. Personalized learning offers methods, learning styles, clearly structured curricula, and school environments that are tailored to the individual needs of students to learn mathematics.

Challenges to personalizing mathematics learning are related to everything related to it: learning outcomes, methodology used, learning techniques and styles, and updating learning resources.

On the other hand, all possibilities must be found to achieve the main goals of learning mathematics for students through the personalization of mathematics learning.

In short, the goals of learning math are presented as shown in Figure 8.

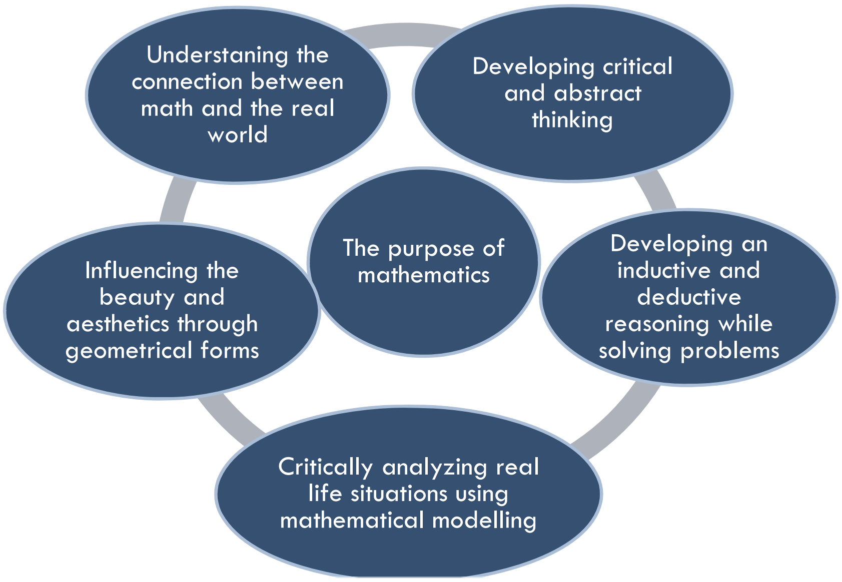


Figure 8.The purpose of learning mathematics

The new basic education is based on a constructivist approach, which states thatlearning occurs when a person builds together the mechanism for learning and his unique version of knowledge, complementing these with his personal experiences and skills. (Roblyer & Roblyer, 2006)

The essence of this approach is the construction of knowledge by the student himself placed at the centre, being facilitated by the teacher and not by dictating the information.This approach on the other hand requires efficient methods to personalize teaching.

Personalized learning in curriculum mathematics through ICTs strongly influences simultaneously in two directions: First**in active learning**, and second it is **a catalyst of the cognitive activation of students**.The mathematical education offered to students goes hand in hand with the natural development of children's skills, which in turn have to do with problem-solving, mathematical communication, collaboration, critical thinking, creativity, reasoning, adaptation, logical reasoning, etc.

ICT tools are the enhancer of personalized learning in mathematics and both together support the range of key skills and competencies that students need to master as future citizens,as shown inFigure 9.

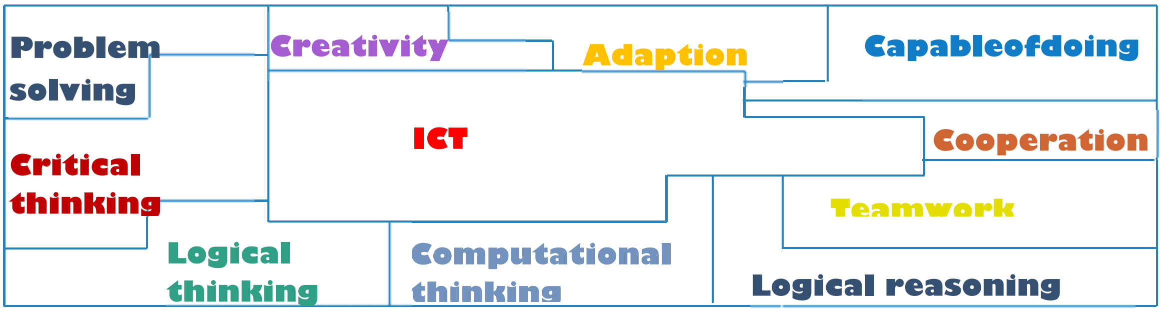


Figure 9 ICT puzzle and children's skills

Personalized learning supported by ICT, will teach students to find the key to opening their gate of reasoning in their way, according to their abilities, their pace, according their needs.Personalized learning should make it possible for the learning process to match students' unique learning styles, backgrounds, needs, and previous experiences.

According to the International Association for Technology in Education (ISTE) (International Society for Technology in Education., 2000)the teachers strongly believe in the role of technology in transforming teaching and learning.

Table 6shows how ISTE standards for students have changed concerning the learning process and the use of ICTs in this process (Brooks-Young, 2016).

Table 7How to have the ISTE Standards for Students evolved

	Year		
	1998	**2007**	**2016**
How have the ISTE Standards for Students evolved?	Learning to use technology	Using technology to learn	Transformative learning with technology

According to ISTE, the following 7 standards have been set for students who use technology in the learning process:

1. Empowered Learner
2. Digital Citizen
3. Knowledge Constructor
4. Innovative Designer
5. Computational Thinker
6. Creative Communicator
7. Global Collaborator (Brooks-Young, 2016)

All of these standards translate into the skills and competencies that students must possess.

Some of them are:

- Using ICTs to personalize their understanding, thinking, and learning;
- Selection of contemporary educational technologies to amplify learning;
- Exploring problems from reality to create ideas on various projects that offer solutions as well as finding the right solution;
- Choosing the right tools to achieve their goals in mathematics;
- Interaction not only with students from their class or school but from the whole community and beyond from all over the world, for the exchange of ideas and solutions to various problems;
- Exchange of ideas or their work during group work for a certain project.

Competent and prudent actions in the digital world have to do not only with privacy policies but also with those of intellectual property.These are very ambitious standards that require the coordination, planning, and implementation of equally ambitious plans and projects of many policy-making structures, which are responsible for education.

3 CONCLUSIONS AND RECOMMENDATIONS

Although personalized learning is defined, explained, and analyzed in different ways, there are some characteristics on which everyone should agree:

- Personalized learning is competency-based learning
- Put the student at the centre of the learning process
- Optimizes the pace of learning and learning approach based on student's abilities and needs
- Orients to move forward with confidence and courage to achieve the final goal
- Supports and promotes logical, critical, and abstract reasoning.
- Encourages creativity and collaboration.
- Develops multidimensional skills that shape the contemporary citizen with fair and rational judgments about the world around him.

The process of personalizing learning is a process that requires continuous and quality commitment. The math teacher, while personalizing learning, performs multidimensional activities at the same time, as is the learning process itself.

Personalized learning in mathematics through ICTs strongly influences simultaneously in two directions:

- firstly, in **active learning**, and
- secondly, it is a **catalyst for the cognitive activation of students**.

The support of ICT tools in the teaching/learning of mathematics is multifunctional. Apart from the fact that it motivates all levels of students, there are many other functions in many aspects of teaching. Let us mention some of them:

1. It significantly improves students' understanding of mathematical concepts;
2. It increases the quality of teaching through visualization;
3. It encourages learning in collaboration;
4. It strengthens student self-esteem;

5. It helps gifted and talented students.

Personalized learning in mathematics through ICT tools, is a process that should be placed as the focus of educational policies for the training of mathematics teachers in Albania, to perform this mission so complex.

ISTE has set very high standards for students, ranging from "Learning to use technology" to "Transformative learning with technology", but achieving these standards requires qualified teachers and ICT tools to support the process.

Some aspects that would take on the current situation in Albania would be:

- The types of ICT tools that teachers have at their disposal for teaching in the schools where they teach.
- Personal perception of the use of ICTs tools in mathematics learning.
- Their professional motivation is to be trained in the use of ICT in teaching.
- Personal experiences with using ICTs tools to personalize learning.
- Difficulties they encounter when using ICTs tools in teaching mathematics.

CHAPTER 3.USE OF ICT IN TEACHING MATHEMATICS IN ALBANIA: STATE-OF-AFFAIRE

We need technology in every classroom and every student and teacher's hand because it is the pen and the paper of our time and it is the lens through which we experience most of our world

David Warlick

Personalization of mathematics teaching means the design, development, and implementation of teaching strategies and techniques based on the skills, and intelligence of each student and is a very complex process that requires the involvement of many actors, responsible for the well-being of education in general.

Nowadays more and more teachers are dealing with the preliminary assessment of the needs and abilities of each student for a lesson based on their strengths as well as the unique desire to implement ICT tools, to personalize teaching. This commitment of teachers to apply technology to the learning process fits in with the growing interest of our students in using ICTs, as well as the range of computer software and technology devices that are changing at a galloping pace.

But do their desires go hand in hand with the digital skills they have? Are they supported by a modern educational technology base in their schools?

What experiences do they have in personalizing mathematics learning through ICT?

The analysis and interpretation of the national questionnaire developed with the teachers of mathematics in secondary schools in Albaniawill shed light on all these issues.

This chapter analyzes the data collected, interprets the findings, and presents the results of focus group interviews regarding their experiences in personalizing mathematics teaching.

1 METHODOLOGY

The questionnaire was intended to collect data on the real situation in Albania based on the support of ICT tools that exist in our schools as well as the digital competencies that mathematics teachers possess to use ICT in personalized teaching and learning in mathematics.

This data will serve us not only to investigate the situation and identify real problems concerning the above purpose but will serve to create policy-initiated models for their improvement.

A mix of methods was used to shed light on the frequency, quality, and variety of ICT used to personalize mathematics teaching.

Instruments:

Questionnaires were used in the first phase and semi-structured interviews were in the second phase.

Regarding the questionnaire, quantitative and qualitative data were collected. The questions were designed to be closed and were mainly of the following types: alternative, quantitative, with multiple choice.

The questionnaire was compiled at Google.doc. and was structured with 33 closed-ended questions placed in the following 6 sections:

1. Sociodemographic information.

2. Professional context.
3. Personal perception of the use of ICT in teaching mathematics.
4. Teachers' interest in being trained in the use of ICT in teaching.
5. Personal experiences with the use of ICT in teaching.
6. Difficulties encountered by teachers in their daily life, to use ICT to personalize the teaching of mathematics. (Appendix 1)

Data collection:

250 questionnaires were distributed in the electronic form to the teachers of the 4 Regional Directorates of Pre-University Education, and another 38 were handed over by hand to the mathematics teachers who teach in secondary schools in the district of Elbasan (from February 15 to March 10, 2021). Of these questionnaires, only 210 questionnaires were completed with all the complete sections.

So the analysis was done on the answers of 210 math teachers in Secondary schools in Albania, who were a significant part of the target population.

Most of the teachers filled in the questionnaire electronically, while the group of teachers at the Local Education Office in Elbasan filled it in on paper. All teachers were provided with anonymity.

The use of ICT in teaching is a topic that has to do with many other mechanisms and factors, and it would probably have been better if the questions were open, where everyone gave their opinion, but the size of 7 pages of the questionnaire could annoy the respondents. Perhaps it would be risky to get a short answer, just to complete the questionnaire, sometimes without proper care and in this form, the reliability of the data would be reduced.

After a preliminary analysis of the data collected from the questionnaire electronically and in person, in a second phase, **semi-structured interviews** were conducted with teachers of the professional network of mathematics in the city of Elbasan in Albania.

The interview sessions **focused on open-ended questions** about the challenges of personalized learning in mathematics, as well as the support that ICT provides in this regard. Teachers bring experiences related to how they personalize math teaching in their schools.

During the interview, it was intended to shed light on the aspects that are not very clear from the answers to the questionnaire received from the teachers who filled it out electronically.

2 RESULTS

2.1 Socio-demographic information and professional context

From the data collected from the interviewees, it was noticed that about 61.6% of the mathematics teachers surveyed belong to the age group 25 – 45 years, which speaks that it is a generation of teachers who are either born in the digital world or are professionally grown with the digital age, thus belonging to the time of the Internet (Figure 10).

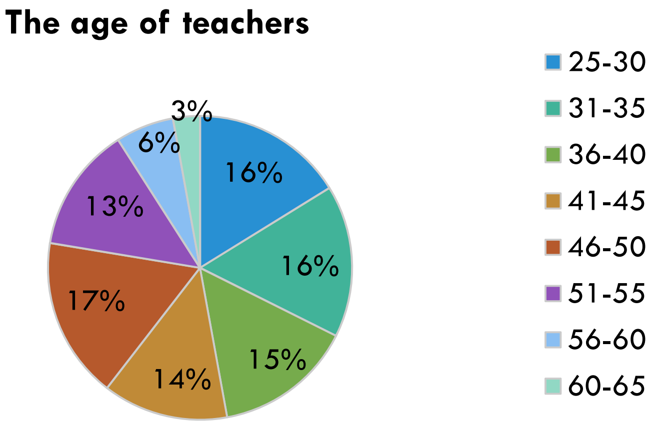


Figure 10 The age of teachers

The Covid-19 pandemic posed the challenge of using ICTs tools in teaching to a group of teachers who, although born far ahead of the Internet and had used technology far less than young digitized people, made continuous efforts to remain competitive in the face of the challenges of technological developments offered by time and to fulfil their mission in terms of online learning.

Javiera Rosell, in March 2021, in his article entitledthat:" Use the Technology – does age matter?" states:*Today we speak about a "digital divide" to describe the differences in the level of technology use by younger and older people. While it is true that older adults use digital technology less than younger generations, many are eager to learn.* (Rosell, 2021)

Of the respondents, 60% are teachers in schools located in urban areas and 40% in rural areas.

The teachers who answered the questionnaires have different work experiences.

Thus, 34% of them have up to 10 years of work, 57.5% have work experience ranging from 10-30 years of work and only 8.5% of them have over 30 years of work experience.

Another important aspect that was noticed from the teachers' answers regarding the professional context is the fact that in 54.8% of the schools the number of students in the class varies from 21 to 32 students, which makes it somewhat difficult to use the ICTs for personalized math teaching in the classroom.

Regarding the weekly load of hours that teachers have, it was noticed that 28.7% of them have less than 16 teaching hours per week, while 30.1% have 16 hours, 35.4% have 20 teaching hours and only 5.8 % of them have more than 20 hours per week.

Understandably, the larger the weekly load, the more commitment and preparation should be made at home by teachers regarding the methods and techniques they should choose to suit the students' level of class, style, and as well as the type of technology to be used.

Regarding the question: "What technological tools are available at your school?" the results presented in the graph below were taken.

The range of technological tools in schools, to be used in teaching mathematics, in Albania is still at minimalist levels.

Although education policies in Albania aim to train teachers with digital competencies, they face a lack of technological tools in their classrooms (Figure 11).

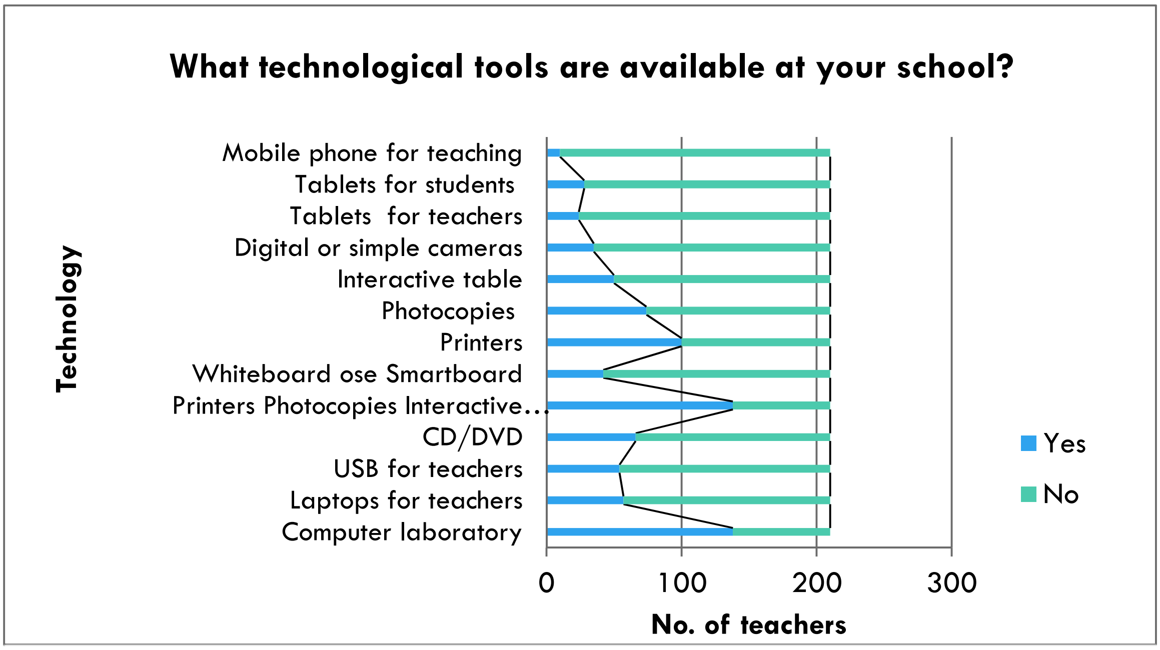


Figure 11 ICT tools in their school

Another aspect that was observed in the study was that27% of teachers had less than 1year of using teachingtechnologies in teaching, only 33% of teachers were 1-3 years old, 15% were 4-6 years old, 8% had 7-9 years, 6% had 10-12 years and 11% had more than 12 years using technology in teaching mathematics.

Knowing that *Instrumental beginnings for teaching mathematics through technology is a complex process that requires time*, (Trouche, Gitirana, Miyakawa, Pepin, & Wang, 2019)and the implementation of technology in the teaching of mathematics in Albania will take its time. This process must be accelerated.

Regarding the question: "What is the approach of teachers and students to technology during the development of mathematics classes in your school?" the data presented in the graph (Figure 12) shows that half of them answered "Never" to "Each student is equipped with a computer with internet access, as well as to " Only the teacher uses the computer and the internet connection to the interactive Whiteboard for the whole class ".

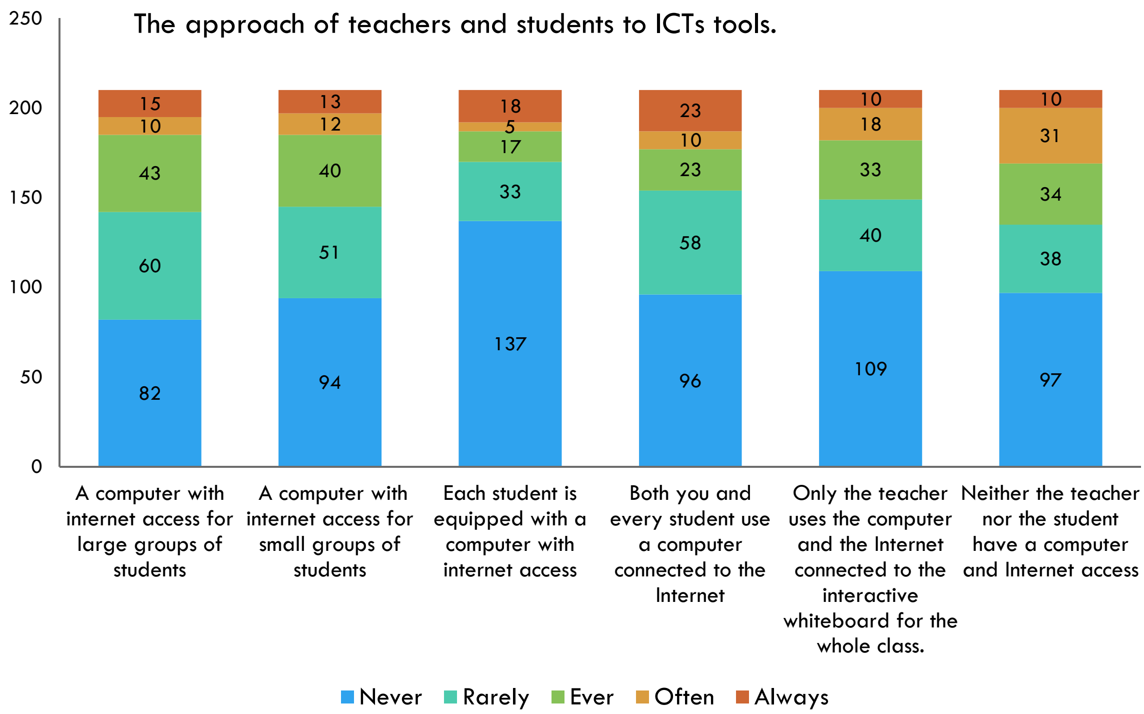


Figure 12 The approach of teachers and students to technology during the development of mathematics classes

From the graphic presentation, it is clear that both teachers and students have proper access to ICTs tools in our schools. Although government policies are investment-oriented to improve the technological base in schools, much remains to be done.

Math teachers are constantly trying to find more student-centred techniques and methods, to make the lessons as attractive as possible and to provide the students with the development they need, according to the capacities they have, but with the ICT s tools that have in their classrooms, in Albania, they cannot do much.

Through educational ICTs tools, they can provide a wide range of tools to access comprehensive sources of mathematical knowledge and collaborate, where they can consult by sharing knowledge and using different sources to solve mathematical problems in real-life contexts.

The data collected from the questionnaire resulted that only 25.5% of schools have a personal domain used by teachers and students, enabling the increase of credibility and access to the school community and giving them an advantage compared to the schools that do not have a personal domain.

Insufficient technological means in schools and limited access to them make students operate within the same day in two different environments. On the one hand, they spend a large time with technological equipment at home, while due to the lack of an educational technological base in schools they do not have access to it.

It is known that students spend time with their mobile phones on various social networks, the purpose is that the time spent with the technological equipment they have has favoured the learning process in general and that of mathematics in particular.

If math teachers carefully orient students towards different platforms, or educational games the benefits would be great, but the results inFigure 13, show that many of them rarely use them.

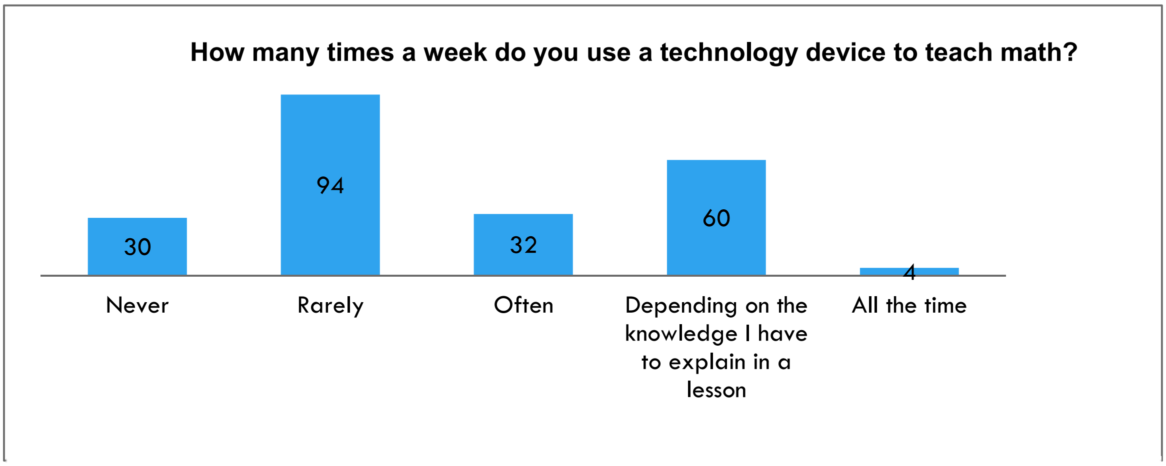


Figure 13Times for a week do they use a technology device to teach math

A careful look at the data in the graph shows that almost half of them use it often or depending on the knowledge they have to explain that lesson, and this is a promising result.

To the question:**Can students use personal technological equipment to benefit the development of the lesson?**, about 69.3% of them answered "only if asked in advance", 18% of them answered "Yes", while 12.7% answered" No ".

Math teachers nowadays are faced with choices that put them in front of professional challenges related to the integration of ICTs tools in the learning process, both in the classroom and outside it. The questionnaire found that approximately half of the teachers use ICTs tools in teaching mathematics. They can use it often or depending on the specifics of the lesson, but they must use it.So, the right technological tool or appropriate software must be implemented at the right stage of the lesson.

When Lagrange addresses the integration of technology in mathematics didactics, he considers four main dimensions: **student, teacher, tool, and mathematics**. (Lagrange, Artigue, Laborde, & Trouche, 2003)

Each of these dimensions has a role to play in achieving the ultimate goal of optimizing learning outcomes in mathematics teaching. It depends on the teacher's perception of when technology implementation is most effective for personalizing math learning.

Regarding this perception, most teachers said that they use technology depending on the mathematical concepts they have to deal with in a class. In addition, a significant number of them say that they mostly use it to demonstrate a learning situation in the first phase of the lesson that has to do with learning anticipation.

Results in Figure 14 show that teachers' opinions vary from use at each stage of the lesson to specific stages to rarely.

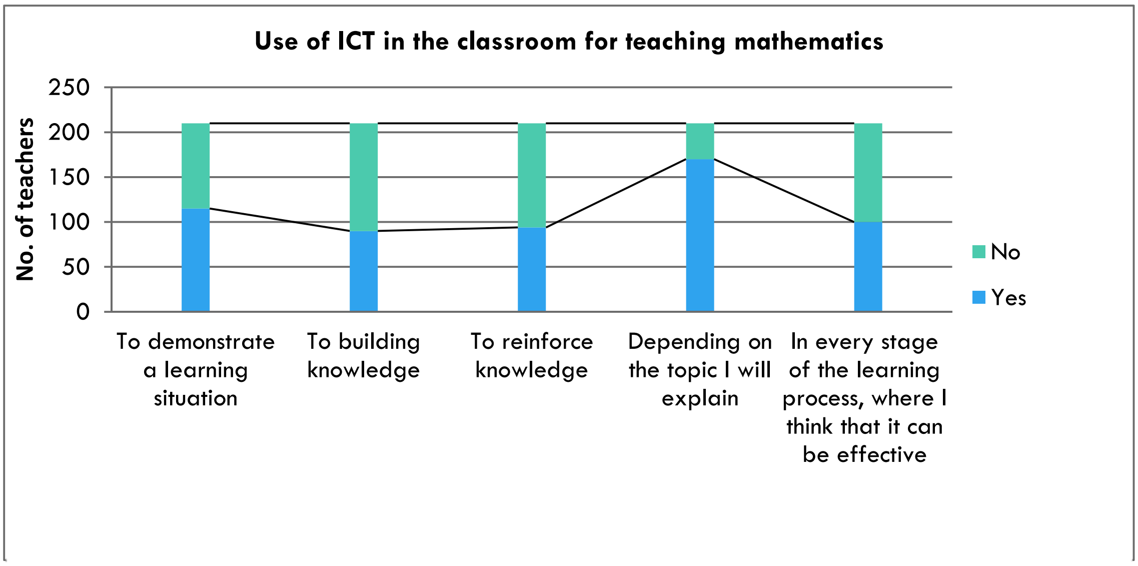


Figure 14Use of ICT in the classroom for teaching mathematics

In the answers given, it is interesting that the opinions about the last alternative "In every moment of the learning process where I think it can be effective" are almost divided in half.

Perhaps such an opinion is related to two main reasons:

1. The impossibility to have the necessary technological equipment in the schools where they work.
2. With a not very high level of competencies in mastering the use of ICTs tools in teaching mathematics.

2.2 Personal perception of the use of ICT in teaching mathematics

For educational technology to be strongly introduced in teaching, a clear perception of teachers on the role of technology in the teaching of mathematics is needed first of all.

They should have clear benefits in the development of abstract thinking (Including the ability to recognize mathematical models, gain experience, thinking skills for mathematical modelling of real situations, and analytical skills that lead to problem-solving), as well as in the development of critical thinking (for example, searching for the simple contrast of a hypothesis or model found, searching for information on the Internet helping to solve a problem, facilitating the understanding of abstract concepts by introducing an abstract concept with concrete examples).

The benefits of ICTs tools in the role it plays in the development of abstract as well as logical and critical thinking are well known (Figure 15).

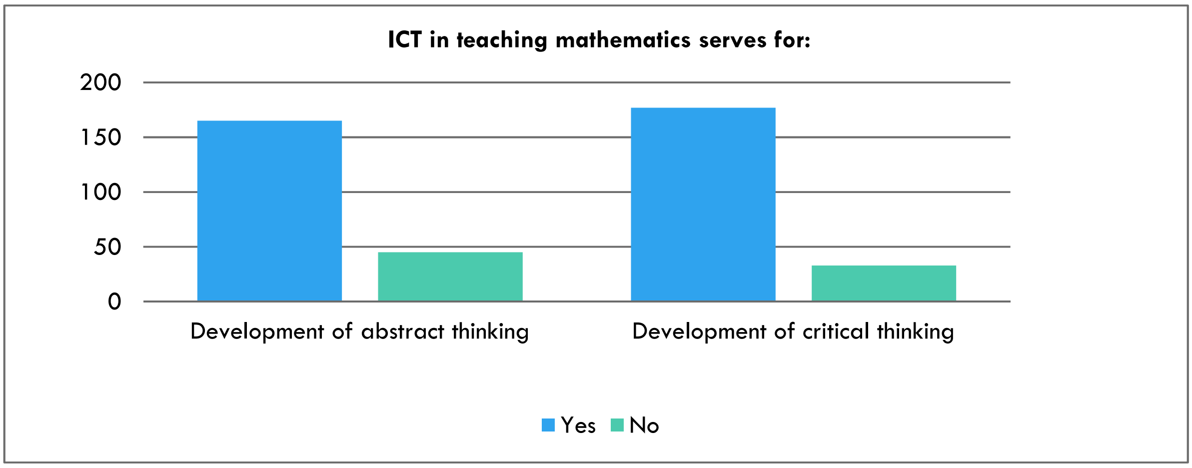


Figure 15 Serves of ICT teaching in math

According to Jones (Jones, 2010), *The two missing elements for many students are the experience of the process andcritically, the reification of the processes into a mental object. The use of software can greatly help here. It easily generates processes with varying parameters and provides dynamic imagery to help achieve reification.*

The results of Albanian students in PISA regarding mathematical competencies, although they are in an increasing trend, should be viewed critically, as the position of Albanian students in it, is a clear indication that our students lack critical thinking skills.

These tests measure precisely the capacity of students to apply mathematical knowledge in the real world and the position of Albania almost at the bottom of the list is an alarming indicator that there is a lot of work to be done with our students with analysis, logical reasoning and effective solution communication and interpretation of mathematical problems.

It should be aimed that gradually, our students start from ready-made models and explain precise scientific arguments concepts, and problems that are subject to much more complex procedures.

According to various research, the wider the range of learning modalities, especially digital learning environments, the more critical thinking is encouraged, and students are educated to be able to apply their mathematical knowledge in the real world.

The whole process starts with access to ICTs tools and follows to the top of the pyramid the path through which the formation of mathematical competencies through ICT passes (Figure 16).

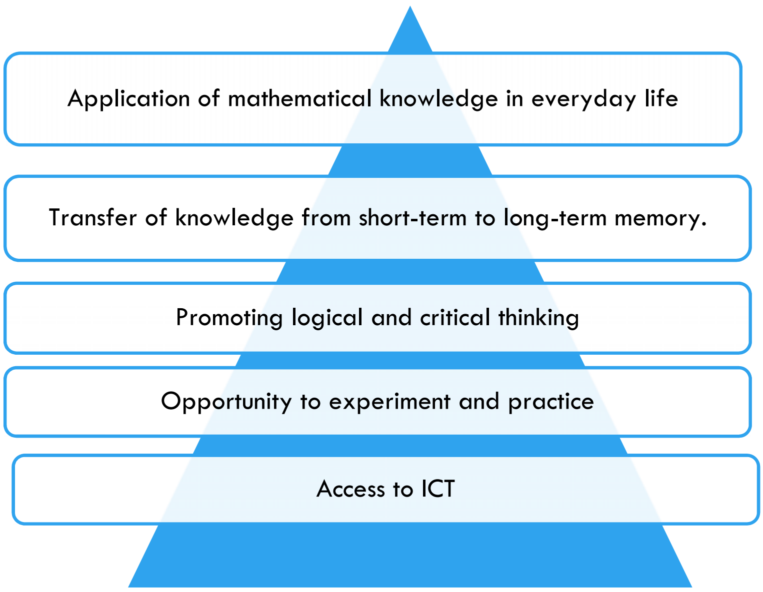


Figure 16Competency building pyramid through ICT

All this process is closely related to cognitive development in preadolescence as well as in early adolescence. At this age, logical thinking tends to progress towards abstract thinking.

Planning skills increase and memory strategies are modernized by increasing long-term memory. Thinking becomes deeper and more critical. Self-regulation and metacognition develop even further. By providing extensive modelling, coaching, scaffolding, and problem-solving, technology offers learners richer opportunities to build metacognitive. (Lajoie, 2008).

In the opinion of mathematics teachers about what the use of ICT in teaching mathematics is for, different teachers have shared different opinions (Figure 17).

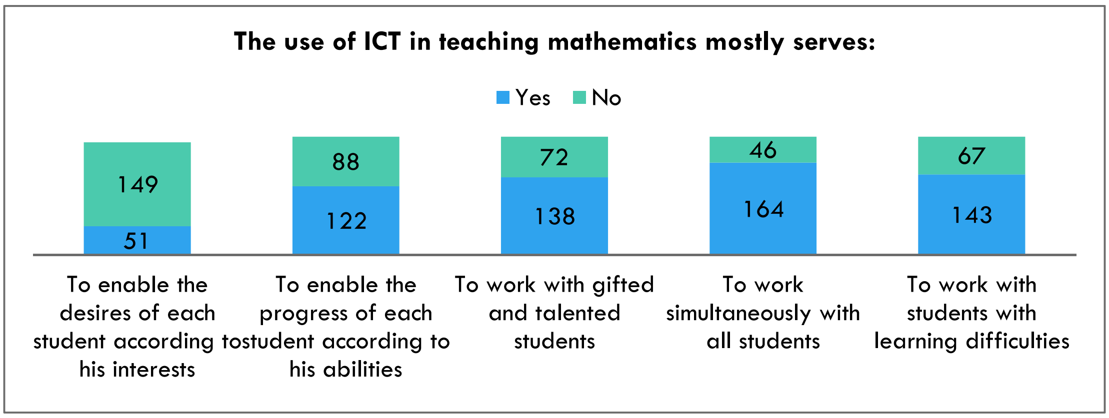


Figure 17 Use of ICT in teaching mathematics

It is noteworthy that a large part of teachers is clear about the role that educational technology plays not only to work with students with learning difficulties but also with those who are gifted and talented or even to enable them to progress according to their abilities.

So, even in the conditions of a very modest technological base, mathematics teachers in Albania see ICTs as a very good opportunity for personalized mathematics teaching.

Opinions of mathematics teachers regarding the role of ICT in the function of personalized learning in mathematics are different.Figure 18 sheds light on their opinions.

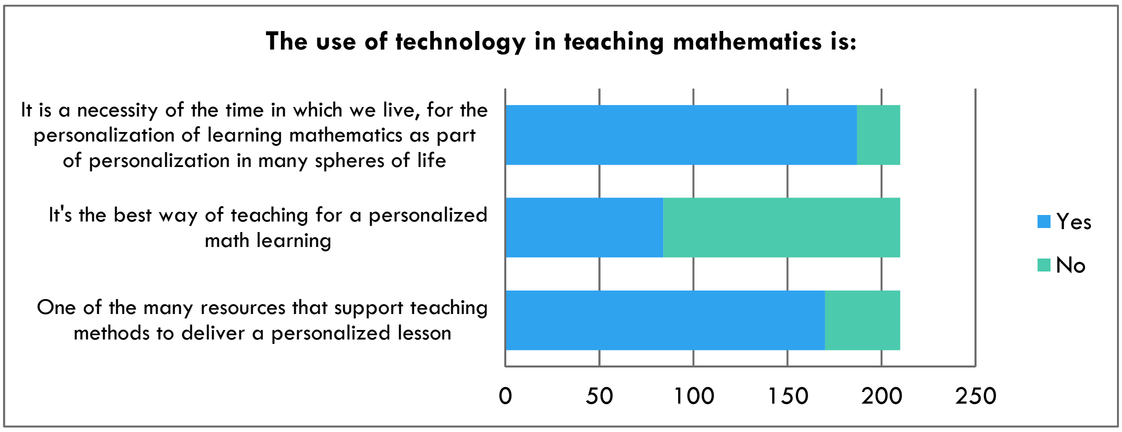


Figure 18 Use of technology in teaching mathematics.

It is to be welcomed that although teachers are aware of the access that they have to educational ICT tools in their schools, about 89% of them see the use of ICTs for personalized teaching mathematics as a necessity of the time we live in, where almost all walks of life require personalized services.

ICT is a magical world where students can search for task information at all levels enabling them to find solutions to similar problems and making them selective in selecting the material or knowledge, that they need to explain the solution to a problem. In this way, ICT becomes at the same time an indicator to educate students in the sense of reflection.

Access to ICTs in the service of mathematics enables students to follow lectures and instructions for types of exercises and problems according to their rhythm. So, students with learning difficulties are offered a series of guidelines to practice, while gifted and talented students are offered additional challenges to progress.

Some of the strengths of personalized learning are:

- Motivating students to act, and not just to follow the teacher's instructions
- Creating planned situations with a focus on developing their practical skills, according to their levels
- Giving time and space to each student, according to his/her rhythm and intellectual mental capacity that he/she possesses.
- Peview their successes or failures as soon as possible

If the above criteria are supported by the *smart* use of technology, quality is guaranteed.

It is no coincidence that almost the same percentage of teachers think that personalized math instruction through educational ICTs tools, is about multi-sensory learning experiences that are based on skills development cooperative and communicative, according to the skills and needs of students (Figure 19).

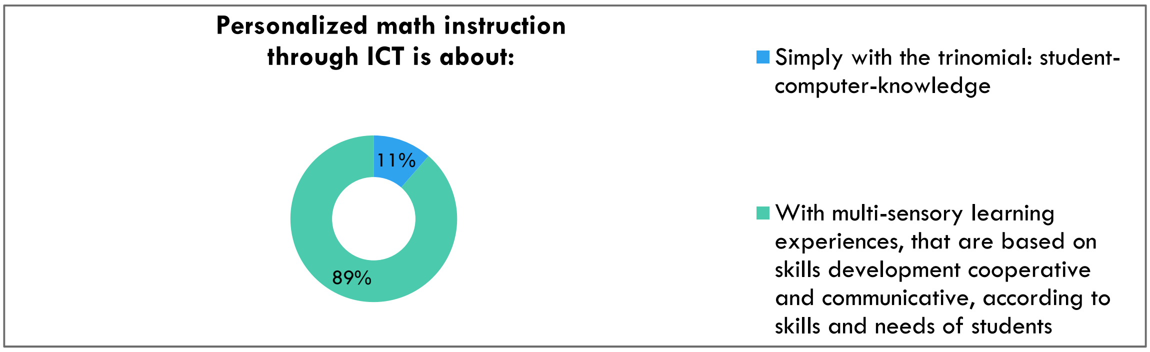


Figure 19Personalized math instructionthrough ICT

It is the same percentage of teachers who think that the use of educational technology in teaching brings a variety of teaching methods and techniques and encourages students to want to learn mathematics. In fact, beyond these benefits, technology enables teachers to differentiate for students the guidance they need to understand and learn mathematics according to their needs and abilities.

The Covid-19 pandemic faced major challenges for teachers in the field of teaching, where many math teachers were forced to use different platforms for online learning as well as provide differentiated instruction with online lessons for each student.

According to (Go Guardian Team, 2020), Education, regardless of the form it takes, capitalizes on a child's natural need to develop skills.At a young age, our brain develops thousands of connections that help to inform our world. These connections have to do with our ability to solve problems, communicate and collaborate. Acquiring these skills is necessary as we need to use them throughout our lives.

This is why all forms and ways must be found to enable the optimal development of students' mathematical abilities and competencies by personalizing the learning of mathematics through the strengthening of technological support.

The pace of technology development is galloping, at the same time increasing the range of technological tools that guide and support math learning for everyone according to their abilities and needs.

In the role of a powerful amplifier to create the skills and competencies required by the future of the world in which we live, technology supports creativity, logical, critical, and abstract thinking, solving various problems from the real world, becoming at the same time a driver of innovation and progress in all areas of life.

In the survey, teachers answered the question: "If the technological base of the school where you work would be very good, what would it be like for you to teach mathematics?"; 57,4% answered that it would be more qualitative, 22.5% did not have an answer, 17.6% thought it would be more personalized, and only 2.5% of them thought it would not change anything.

It is understood that the number of sceptics about the use of educational technology in teaching mathematics is very small, but what stands out is the high percentage of teachers who do not have an answer about the role that technology can play.

This is a fact that should be taken into consideration by education policymakers to provide qualifying training on the role of technology in teaching mathematics.

2.3 Teachers' interest in being trained in the use of ICT in teaching mathematics

In the Albanian education system, there is a lack of a stable infrastructure in the field of ICT in teaching and digital learning resources, which have an urgent need to increase investment in technological tools that can be used efficiently in schools.

On the other hand, this problem goes hand in hand with the need for mathematics teachers to develop high ICT pedagogical competencies in teaching.

By providing, firstly, a sustainable infrastructure and, secondly, ongoing training of teachers to master digital competencies in teaching, we also provide students with a comfortable development of digital competencies which are so much needed in the world today and are important for competent learning in the 21st century.

According to (Vegas, Ganimian, & Jaimovich, 2012), *an education system is as good as its teachers. Decomposing their potential is essential for the quality of learning.*

The teaching profession is a profession that must undergo continuous training, from others or self-taught, not only to be in coherence with galloping technological developments but even to advance them.

In Albania, teacher training is provided by licensed training agencies regulated by state education policies.

The training focuses mostly on the implementation of the competency-based curriculum, teaching methodology, student assessment, learning portfolio, approaches to technology, etc.

In the situation created by the Covid-19 Pandemic, the training of teachers in the field of ICT was dictated as a necessity, mainly in virtual teaching and the use of platforms and applications related to teaching in general and teaching mathematics in particular.

The chart below (Figure 20) shows data on the types of ICT training provided to math teachers.

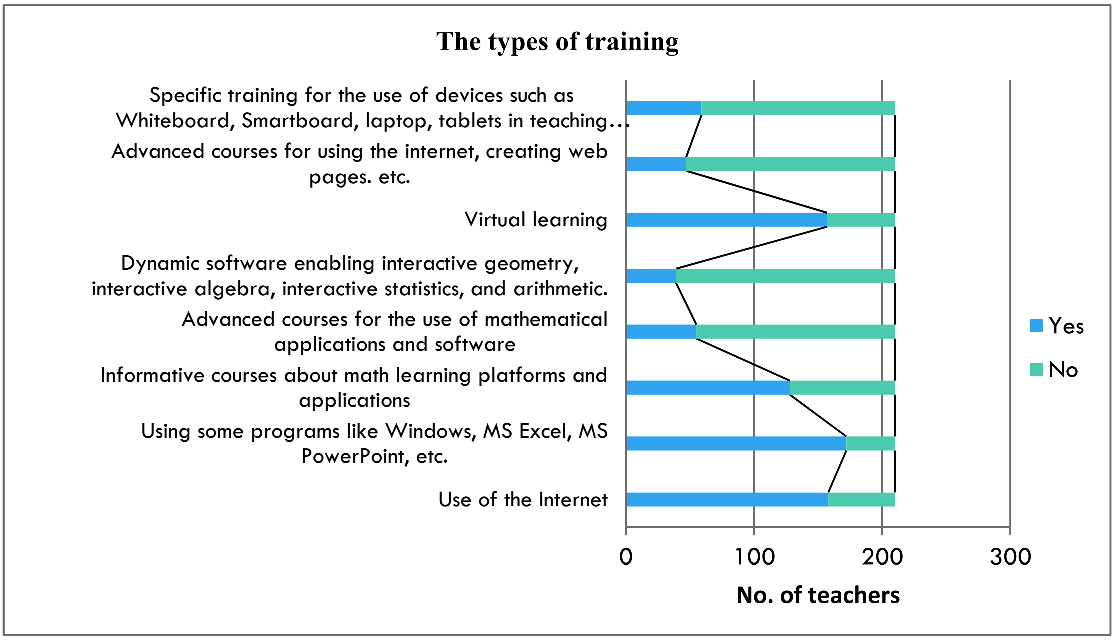


Figure 20Types of training provided to math teachers

The data showed that virtual teaching, the use of the Internet, and the use of basic computer programs are the training that the interviewees have developed the most.

A little over half of them have taken courses related to applications and platforms that support the teaching of mathematics. Around 25% are trained in the use of the Whiteboard, SmartBoard, laptops, and tablets in teaching.To the same extent, they have taken specific advanced courses dealing with dynamic software including geometry, algebra, interactive statistics, or in general for the use of mathematical applications and software. A high number of them, 82%, have attended courses for the use of basic software such as Windows, Excel, and PowerPoint, while about 75% of them are trained to use the internet.

The number of teachers trained in the use of mathematical applications and software is still minimal.The nature of the training developed by mathematics teachers varies from those organized

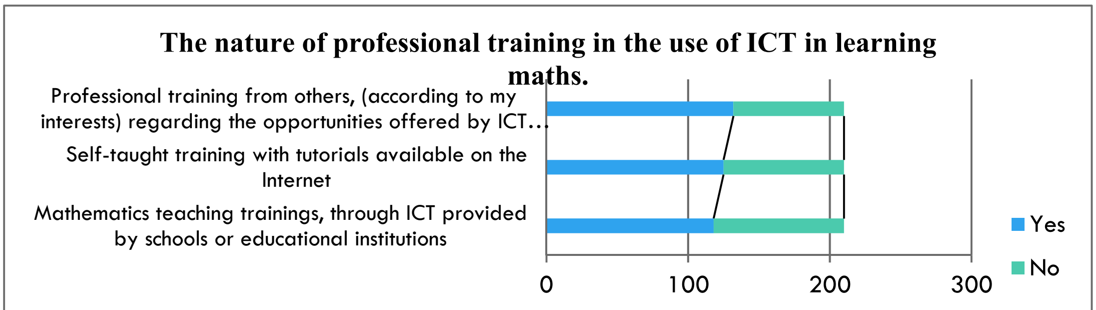


Figure 21Nature of professional training in the use of ICT by educational institutions, to self-taught training with online tutorials, as well as to specific training developed at their request, for teaching. Figure 21 sheds light on the types of training.

As for the number of training in the field of ICT provided by the education authorities in the last 5 years, the situation is as follows inFigure 22.

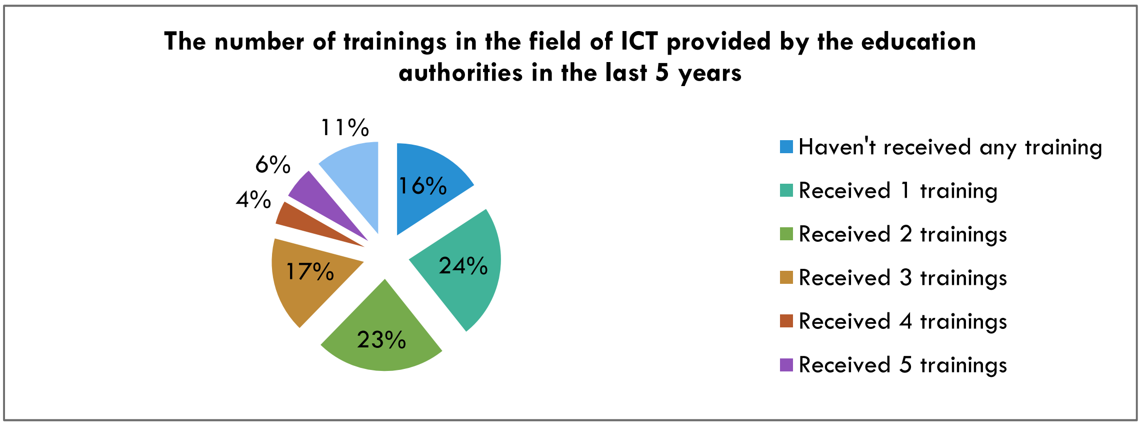


Figure 22The number of training in the field of ICT provided by the education authorities in the last 5 years

Math teachers need to be demanding of themselves, to adapt technology to the classrooms where they teach. All they have to do is start training by finding different ways and not just waiting for the relevant educational institutions. There is a huge range of tutorials for using technology in teaching, which is offered for free online. The only problem may be the mastery of the English language, as the tutorials offered in Albanian are limited.

Teachers need to transform classrooms into innovative learning environments, where each student is strongly supported by technology-based methods and techniques, which develop their potential and make mathematics learning attractive.

Through technological means, teachers should find ways to be a powerful transformational support to the traditional lessons they have developed so far, supporting new methods that personalize the teaching of mathematics through active learning methods and the student's mental and intellectual capacities.

The chart inFigure 23shows the data on the organization of training, the persons responsible for them, as well as whether the school institution offers training in the field of ICT.

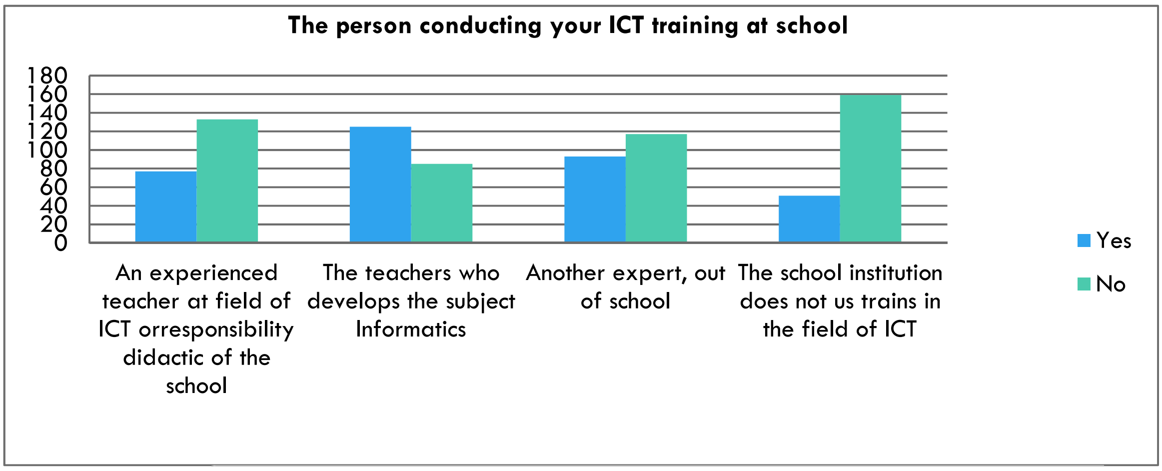


Figure 23The person conducting your ICT training at school

It is noticed that in most schools, the specialist who also covers the training in the field of ICT in teaching is the Informatics teacher in that school, an expert specialist coming from outside the school, or a teacher with experience in the field of ICT.

Only 51 teachers responded that the school institution does not train them in the field of ICT.

Ongoing training in the field of ICT requires the coordination of state institutions following the needs of teachers to be trained in the use of technology in teaching.

On the other hand, teachers should be aware that the inclusion of ICT in teaching and learning is not a process that has its beginning, its completion, and its end.

It is a process that lasts and improves in parallel with technological developments, a process that is constantly updated, and that requires in the first place the attention and interest of mathematics teachers to undergo this training.

Perhaps it would be important for teachers to be required to provide a certain amount of credit for the training they receive concerning training in new ICT-assisted teaching methods.

The professional development of math teachers in the field of ICT is an immediate task of our education system.

Although educational policies guide the importance of ICT-related training, teachers need to learn from each other and share in their professional networks positive and in fruitful experiences.

Students often have ICT knowledge that is admirable and can serve as a powerful support for teachers in their classrooms.

With good classroom management and wise division of roles in a classroom, the math teacher can engage talented ICT students, increasing their motivation and developing self-confidence.

Good experiences are also those where members of the parent community, who may be ICT specialists, can provide training or assistance to math teachers to support the implementation of ICT in math teaching.

Thus, the benefit is twofold, students develop quality lessons based on technology, while the teacher develops professionally in the field of ICT, with the support of the expert.

For the development of competencies and skills of teachers in the field of ICT, the range of actors who need to coordinate their efforts is very wide, containing school directorates, local and regional education directorates, and professional development networks of mathematics teachers.

The wider the spectrum of actors supporting technology-supported personalized mathematics teaching, the faster the process of its implementation in teaching, and the more rapidly its positive role in teaching will change the study of mathematics.

However, regardless of the actors who may be involved in accelerating this process, education policies in the Ministry of Education must have well-oriented, coherent, and easily implemented program plans.

2.4 Personal experience with the use of ICT in the classroom

Based on the modest technological base of their schools as well as their computer skills, the periodicity for the various activities that support teaching is shown in Figure 24.

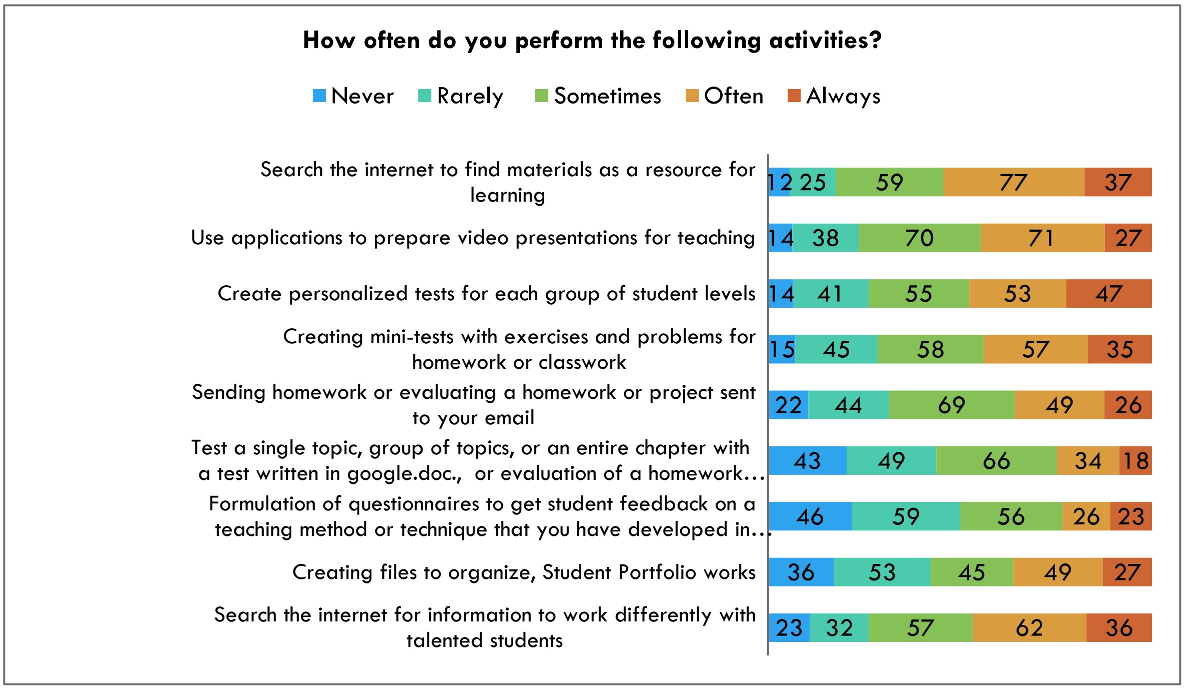


Figure 24 Periodicity of activities with ICT in math

The question:"*How often do you perform the following activities?*", was addressed to math teachers to investigate how present are some activities based on the use of ICT in mathematics teaching and how often they are performed by math teachers.

What is striking and unfortunate is the fact that about 6% of respondents never perform an activity related to the use of ICT in mathematics teaching

This percentage is even higher when it comes to testing groups of learning topics or knowledge of a particular chapter or creating folders to organize student portfolio work.Regarding the materials that teachers use to teach mathematics in the classroom, it was noticed that (Figure 25).

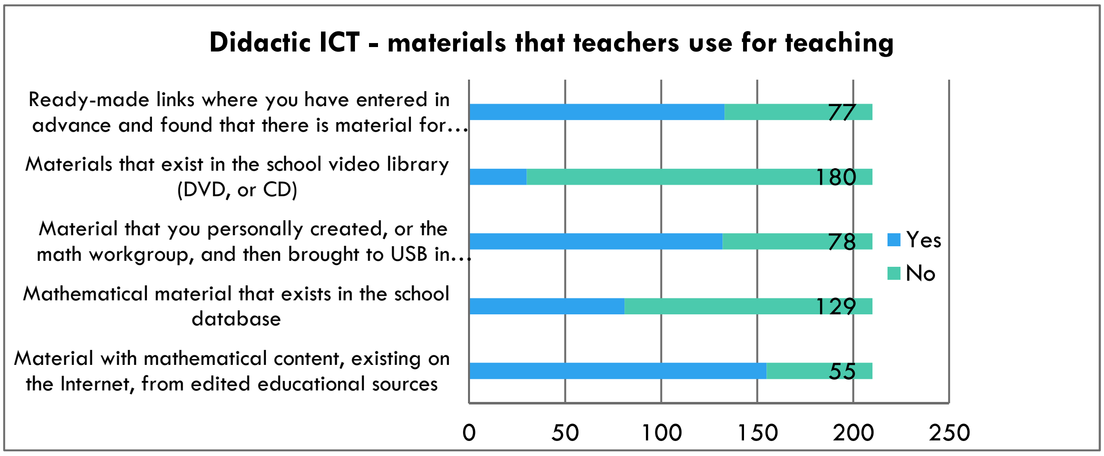


Figure 25 Didactic ICT materials that teachers use for teaching math

- 74% of them use ready-made didactic materials taken from edited sources on the Internet and use them in their classes.
- 38.6% use materials with mathematical content that exist in the school database.
- 63% of them use materials that they have previously created at home, themselves, or the professional network of math teachers, and have brought to school via USB.
- 14% also use materials that exist in the school video library.

What stands out is the large percentage of teachers (63%) who use ready-made links that have been checked in advance and that contain material that personalizes math learning.

While on the question: *Which of these Digital Management Systems do you know how to use?* found that more than 50% use Googleclassroom (https://classroom.google.com/), and more than 83% know how to use Akademi.al (https://www.akademi.al/), while 14% use Edmodo (https://new.edmodo.com/).

Only a very small number of teachers can use CenturyTech (https://www.century.tech/), Ekstep (https://ekstep.org/), or ClassDojo (https://www.classdojo.com/), etc.

A high percentage of teachers can use Digital Management Systems, such as Google Classroom, and Akademi.al is related to the fact that at the time of Covid-19 when online learning was applied, these two systems were the most used in Albania.

Questionnaire results on the question: *Which of this software have you used to personalize your math lesson?* are presented in Figure 26.

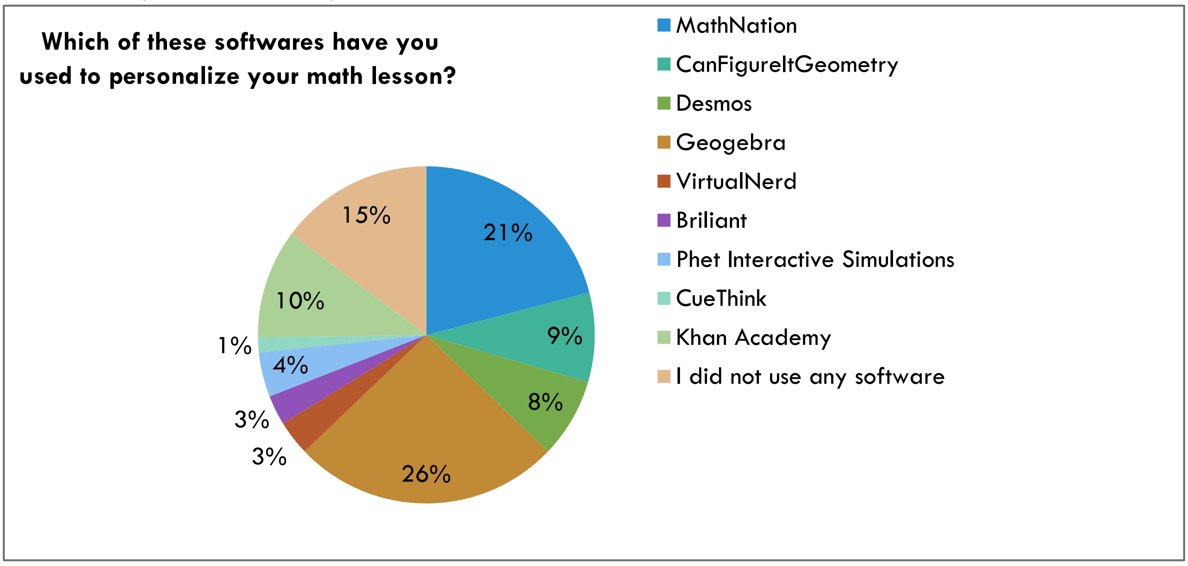


Figure 26 Software usedto personalize math lessons

About 1/3 of the math teachers interviewed stated that they used GeoGebra in teaching mathematics, while about 1/5 of them used MathNation. From the answers collected concerning this question, the fact that it is matter of concern is that 15% of teachers do not know how to use any software, at a time when mastering digital competencies have become a necessity of the time.

Referring to a UNESCO document (UNESCO, 2011) where the competencies related to the educational technology that 21st-century teachers need to master are stated:

- Operate computers and use basic software for word processing, spreadsheets, email, etc.
- Evaluate and use computers and related ICT tools for instruction
- Apply current instructional principles, research, and appropriate assessment practices to the use of ICT.
- Evaluate educational software.
- Create effective computer-based presentations.
- Search the Internet for resources.
- Integrate ICT tools into student activities across the curriculum.
- Create multimedia content to support instruction.
- Create hypertext documents to support instruction.
- Demonstrate knowledge of ethics and equity issues related to technology.
- Keep up to date as far as educational technology is concerned.

The use of technology in the teaching of mathematics creates great facilities for the creation and collection of materials of a didactic nature. It is very important that educational technological changes that occur at a very fast pace be welcomed with interest and assimilated by teachers while evolving in parallel with the changes that undergo curricula, teaching methods, and techniques.

Gone are the days of a teacher transmitting knowledge in the classroom, the teacher of modern times helps to build knowledge based on technological approaches and create appropriate learning environments that put the student in real-life situations.

2.5 Teachers' opinions about the factors, that affect the difficulties they have in using ICT in teaching mathematics

Regarding the question: "*Indicate to what extent the following factors negatively affect a personalized teaching of mathematics through ICT?*" teachers have divided opinions as shown in the graphs.The difficulties that teachers encounter in personalizing mathematics teaching through ICT range from parents' scepticism about learning math through ICT to a limited amount of teaching platforms related to mathematics, translated into Albanian (Figure 27, Figure 28).

What is striking is that most of them see the greatest difficulty concerning the low budget that schools have to create digital classrooms and provide technological tools to use in teaching (Figure 27).

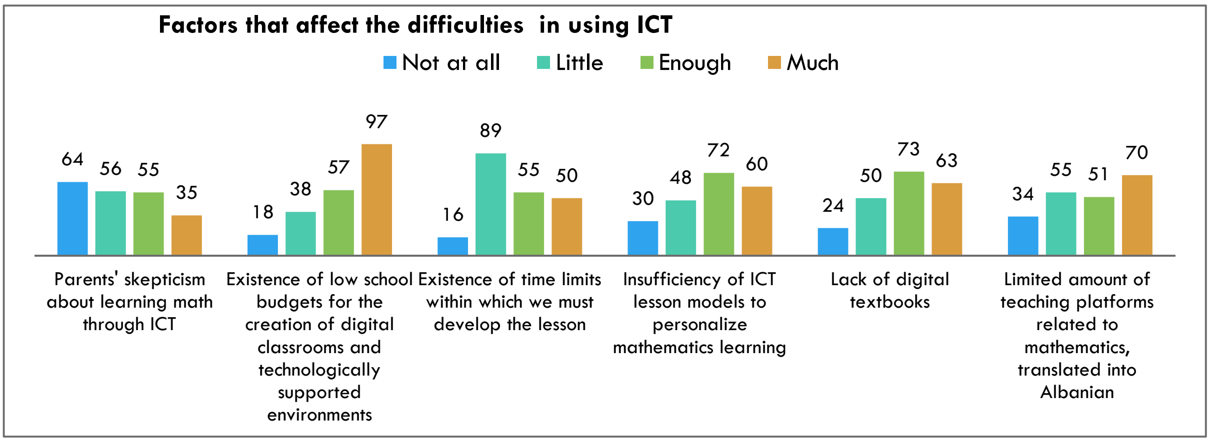


Figure 27 Factors that affect difficulties in using ICT

Undoubtedly, this is a very important factor, but if a teacher has a strong desire to implement technology in teaching, he finds ways for it to work.

On the other hand, more than half of them associate the difficulty of personalizing mathematics learning with the limited number and poor technical condition of computers in their schools, as well as the lack of SmartBoards and Whiteboards (Figure 28)

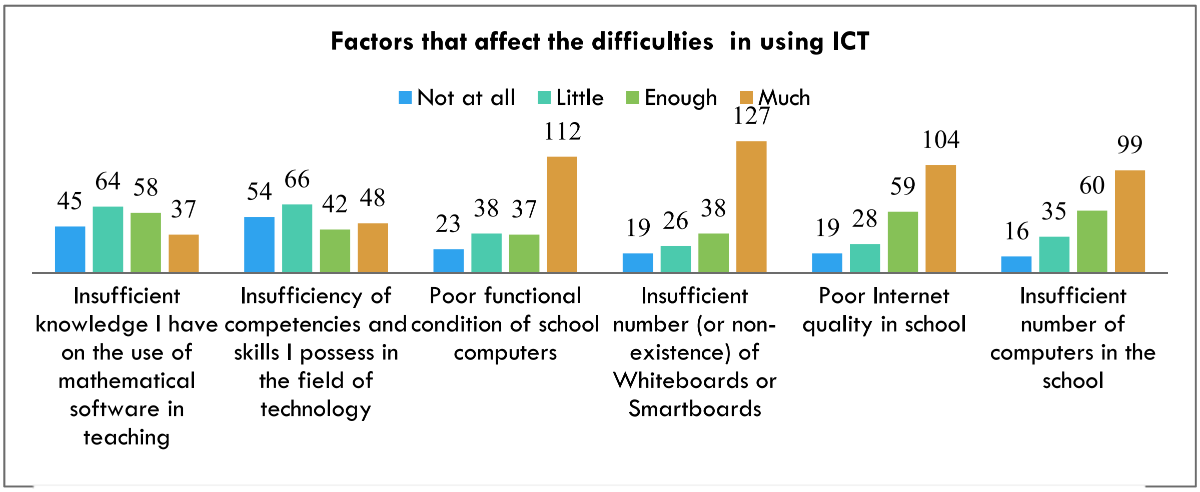


Figure 28 Factors that affect difficulties in using ICT (part 2)

This is indeed a very big barrier, but the careful management of all technological resources as well as other auxiliary resources can alleviate this barrier to some extent. Mathematics teachers also answered the question: *How well do you master the activities related to the use of ICT in learning mathematics?* (Figure 30, Figure 29)

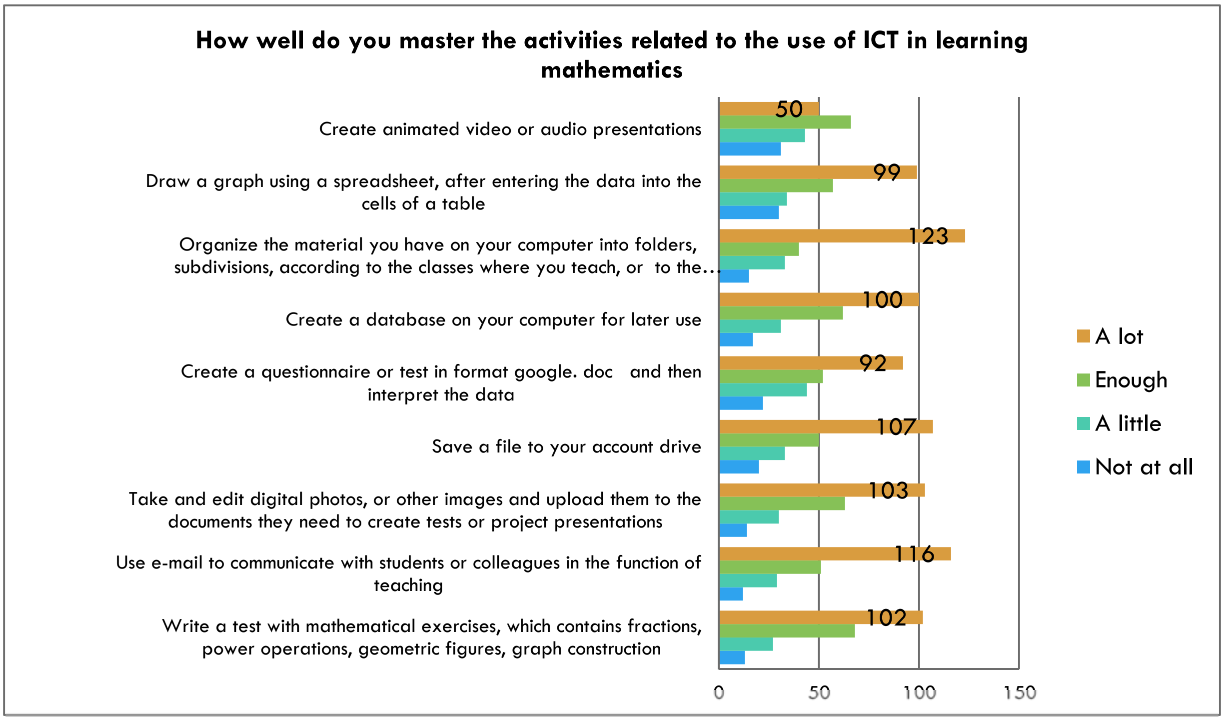


Figure 30 Mastering the activities related to the use of ICT in learning mathematics (part 1)

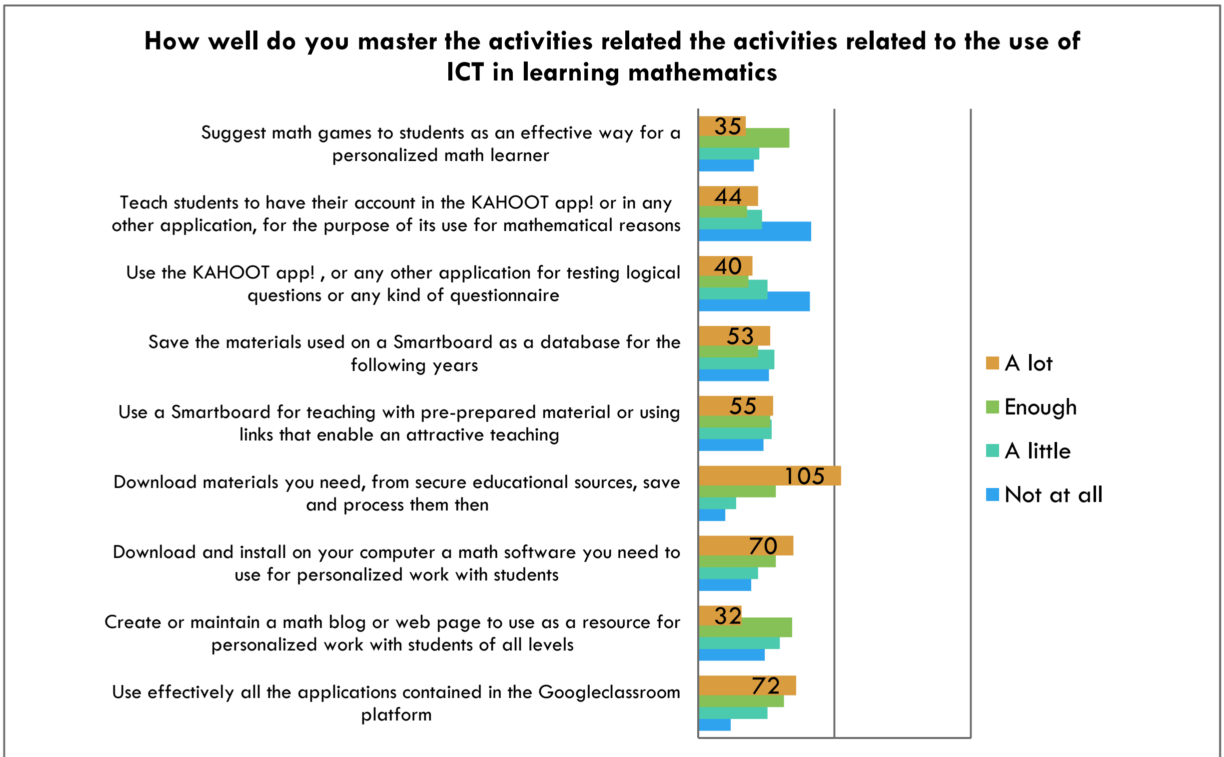


Figure 29 Mastering the activities related to the use of ICT in learning mathematics (part 2)

Among the alternatives given to the digital skills and competencies that a math teacher should possess for quality teaching/learning, it is noticeable that most of them possess at an intermediate or very good level, the basic skills, and competencies such as:

- Writing a test, which contains exercises that contain actions with fractions and powers, different geometric and graphic figures.
- Using email to communicate with students or colleagues across professional networks for teaching purposes.
- Making, editing, and modifying digital photos, or other images and uploading them to the documents they need.
- Creating a database on their computer for later use.
- Creating a questionnaire or test in google doc format and compiling and then organizing the data.
- Saving a file to their account drive.
- Systematizing the material on their computer into folders, subfolders, etc.
- Drawing a graph using a spreadsheet.

On the other hand, it is almost 50% of teachers possess slightly more ambitious skills and competencies such as:

- Creating an animated video or audio presentations
- Using effectively most of the applications contained in the google classroom platform
- Creating or maintaining a math blog or web page, to use as a resource for personalized work with students of all levels.
- Downloading and installing, math software they need to use for teaching on the computer/ mobile device.
- Downloading material from secure educational sources, saving and processing it later.
- Using a smart board for teaching and learning with pre-prepared material.
- Saving the materials used on the SmartBoard as a database for following editions.
- Using KAHOOT! app or any other application for testing logical questions or any type of questionnaire.
- Teaching students to have their account in a specialized application, to use it for mathematical reasons.

The graphs in Figure 29 and 30, shows that teachers possess the basic skills that have to do with the use of educational ICTs tools in teaching.

It is noteworthy that few efficiently use platforms for teaching or learning games in math classes. Very few of them suggest math games to their students as an effective way to personalize math teaching, and just as few can install the software they need to personalize working with their students.

Math teachers need to replace static lessons with gamified lessons, ensuring that all students are involved and keeping them motivated and focused to advance at their level.

Almost half of the teachers possess little or not the majority of the basic skills and competencies they need to carry out a productive learning process.

3 Analysis of the interviews of mathematics teachers regarding personalized learning and the role, that technology plays in its realization

As mentioned at the outset, the second part of the study on the support that ICTs tools provide to personalize math learners were semi-structured interviews with math teachers who said they had experience using ICTs tools for personalized teaching. Interviews were conducted with math teachers in the city of Elbasan after completing the questionnaire.

They expressed in different ways how they conceive of personalized mathematics teaching and how they personalize mathematics teaching in their classrooms with the help of technology and how necessary they are for training on improving digital competence in mathematics teaching.

3.1 Methodology

Completion of the questionnaire by hand by the teachers of the professional network of mathematics in Elbasan was done specifically to conduct some semi-structured interviews with 8 teachers.

A qualitative information-gathering strategy was used, where respondents were asked a series of open-ended questions which aimed to get their opinion on how they conceived those personalized mathematics teaching in their schools and what they were efforts they made to personalize the teaching of mathematics, in terms of the technological basis offered by the school. Then it was discussed about training topics that for them would be very necessary for the current conditions. It was agreed that for the needs of the mathematics teachers of the professional network of Elbasan, training would be very necessary regarding the role that technology plays to support the learning of mathematics according to their learning styles and the use of GeoGebra, SmartBoard, and the Kahoot!

The main questions where the interview focused were, among others:

1. The difficulties you encounter in your work are related to:
    a. the mathematics curriculum;
    b. the quality of textbooks;
    c. students' interest in mathematics;
    d. the difficulties you have adapting to ICTs in the classroom;
    e. lack of training;
    f. all these above;
    g. other factors
2. Share an experience from your work where you think you personalize learning math. What ICT tools do you use in this process?

Below are some of the opinions collected by teachers in the district of Elbasan interviewed on their personal experiences in personalizing the teaching of mathematics in the current conditions of Albania.

Teacher A.

> *Gone are the days when the teacher had only his chalk and his mind, and with a single tool he was obliged to give everything. Now the range of teaching resources has been added. Students have the opportunity to access platforms and software, interactive games with a mathematical characters to build a certain mathematical concept in the*

most fun and in the most personalized way. I always take care to be up to date on the innovations that math education sites offer on the Internet and to suggest them to my students.

Teacher B.

The time it takes for a teacher to understand if a concept or set of concepts has been assimilated is now just a click away. I use quizzes pre-designed on Google.doc, which help me to verify if new knowledge is assimilated by the students and to create plans to identify each other's shortcomings. Along with the shortcomings and the set plan, you decide to include a technological tool or a platform or website which will help to achieve the set objectives.

Teacher C.

I often apply one of the active, problem-based teaching approaches, also known as problem-based learning. I pose a problem in advance, quickly research what the weak points are for each, and then personalize, based on the needs of each student, the lessons that I will develop in the following classes. I have already made a pact with my students, they know that I will try to give them assignments, even in those "areas" where they do not feel comfortable, so and do not hesitate to openly express their ambiguities. I know they like to browse the internet and I often suggest math pages where they can practice. One site I use often is https://www.matematicamente.it/.

Teacher D

At the beginning of each chapter that deals with a mathematical topic, I compile a document in Word with a two-column spreadsheet. In the first, I set the learning outcomes for that topic by scaling the levels, from the basic level to the talented and gifted level, and in the second I set the types of exercises and problems that belong to each level.

For me, the biggest challenge of personalized learning is the rigorous fulfilment within the time frame of all the tasks given to students to work in a personalized way. Giving these exercises and problems, according to a pyramid of difficulty levels, allows students to jump to a higher level immediately after completing a level, always having something more to practice. All the exercises from the base to the top of the difficulty level pyramid, I put in my account in Kahoot!, and the students have access to it so they can go to the computer lab and practice there in the "Kahoot! application! ". There I have many variants of tests because I have deposited year after year, exercises of different levels of difficulty. For them, it is an activity that they do more willingly, as they also learn math while playing.

Teacher E

Instead of waiting to test the knowledge that students have gained at the end of the chapter, I test the level of knowledge they have about the chapter that they will begin. Everything about personalization starts with the assessment of prior knowledge because this assessment allows me to adapt the instrumentsto each student and to choose the methodology and strategy appropriately. This process begins with the preparation of a preliminary test on Google. Doc., with scalable knowledge about the concepts that *students from the previous class should possess, and immediately after that I organize my work for each student to meet the final goals.*

To my follow-up question what does it mean "organize my work" he replied:

By organizing work, I mean a series of activities in the learning process, so that the learning process slowly aims to be directed by the student. This process normally takes time, but if we motivate students constantly, by organizing activities with clearly defined goals and details, and by making active participation, students are also on the right path to achieving success in mathematics.

Teacher F

I often allow students to browse the Internet by being served several math pages or platforms in advance. For this, I use the computer laboratory. It is noteworthy that there, the students are all motivated to work according to their pace and abilities. They often resort to different methods to challenge themselves. There are various computer programs on the Internet where I can identify the ambiguities and weaknesses of each student. After keeping specific notes for each student, I personalize my work with them. Even in the instructions, I give them beyond school hours, I make sure that they are as diverse as possible, to give space to all their abilities.

Teacher G

In our school, we have divided the work to personalize the learning of mathematics. We as a subject team, study in advance each theme of each class, and then design in detail activities for students of each level. This database with exercises and problems for topics of different levels is a valuable asset and we use and enrich it every year.

Teacher H

To personalize math learning, I often stimulate the students' ability to work independently. This does not mean letting them work alone, or in small groups, but encouraging them to practice investigating and discovering based on technology. To avoid the chaos that may arise in this case, I structure all this practice well, clearly setting out the expectations for each, and guiding each student to what he or she expects from him or her, and what he or she should do.

3.2 Results of the interviews with math teachers

In the opinion of the interviewed teachers, to personalize the learning of mathematics by using ICTs tools, it was noticed that they can personalize the learning of mathematics if:

1. Take care to be in coherence with the innovations offered by technological developments in the field of mathematical education.
2. Assess the preliminary situation of the level of knowledge that students have with simple quizzes designed on Google.doc
3. Orient students' learning towards active learning methodologies.
4. Challenge themselves to complete on time the tasks they have planned to personalize the work with each level of students.

5. Investigate the skills and competencies that students possess for a learning topic or group of topics through quick quizzes on Google.Doc. at the end of the lesson, and set plans as well as platforms or software that can be used for improvement.
6. Orient students to browse the Internet in math tutorials where everyone feels comfortable learning at their own pace.
7. Work in teams, to create databases with exercises and math problems according to student levels, in the subject team of math teachers, and exchange them with each other.
8. Practice students' independent work encouraging them to investigate and discover based on technology.

The answers received from the interviews of mathematics teachers, who brought their experiences related to personalizing mathematics teaching through ICT tools, clearly show that in Albania there are positive examples of teachers who are interested in using ICT tools. to personalize math teaching and they are innovative and creative in bringing a range of techniques and methods to make teaching as productive as possible.

What remains to be done is continuous training for teachers, to keep pace with technological developments in the field of education.

3.3 Conclusions and recommendations

ICTs are a powerful tool in supporting the process of personalizing mathematics learning by strengthening its cognitive processes.ICT tools in support of personalized learning enable each student to develop his/her intellectual potential in an unique way of learning mathematics.

A reform to move away once and for all from traditional ways of teaching/learning must be essential.

Based on competence learning, where the learner should be at the centre, teachers should embrace active, technology-supported learning methods, moving from behavioural theories to active learning theories. On the other hand, they need to apply curricula and active teaching methods that enable students not only to develop mathematical competencies but also to use them. In this way, they will once and for all get away from teaching that is based on unconstructive content.

It is important that the use of ICT tools in teaching mathematics with a focus on the student and the development of critical and logical thinking, not face the scepticism of the teacher, but be seen as the right tool leading to a constructivist approach, the Approach constructivist orientation towards lifelong learning competencies, fostering creativity, critical reasoning, and logical reasoning.

Lesson time is not only the time, or physical space available to the math teacher, but it is also the moment when the formation of mathematical concepts comes to life, the formation of students' mathematical culture for life.

From this point of view, this environment should be complemented and treated carefully not only from the point of view of educational technology support but also from the point of view of planning.

On the other hand, the planning for each lesson should be done carefully, it should be constructed and planned in such a way that the learning outcomes for each student are optimal.

In this planning, care must be taken to find the right moment, where the intervention through ICT tools brings to each of the students, depending on the learning modalities and his learning profile, the maximum benefit.

The results of the questionnaire shed light on the training needs of mathematics teachers in Albania as well as the access they have to ICTs tools in the schools where they work.

From the analysis of the answers, given concerning the questionnaire, what stands out is the fact that the variety of ICT tools available to mathematics teachers in the schools, where they work needs improvement. They have technological devices in use in everyday life, such as mobile phones, laptops, or personal computers, but only through them they can not properly carry out an entire process such as active teaching through technology.

Half of them use ICT tools rarely or not at all in teaching and this is an indicator that should be considered by education policymakers, to create policies that firstly offer educational technology-based schools and secondly, train teachers in the use of ICT.

This process should not be seen as a sporadic process in specific schools or certain periods, but as a whole ongoing process in the form of institutional policies organized with detailed programs.

On the other hand, in the framework of these policies, it should be defined as an obligation of mathematics teachers their self-training regarding the digital skills and competencies that they should possess, according to their needs, at the licensed training agencies that exist in Albania.

The planning for each lesson should be done carefully, it should be constructed and planned in such a way that the learning outcomes for each student are optimal.

In this planning, care must be taken to find the right moment, where the intervention through ICT tools brings to each of the students, depending on the learning modalities and his learning profile, the maximum benefit.

CHAPTER 4. TRAINING TEACHERS TO PROVIDE PERSONALIZED MATH EDUCATION, SUPPORTED BY ICT TOOLS

If we teach today, as we taught yesterday, we rob our children of tomorrow.
John Dewey

A teacher who has just finished his studies and is licensed as a mathematics teacher has just started to learn and discover about an ever-changing and perfecting "world" such as teaching. He must be in coherence with all the positive changes that improve his work as well as with the changes in the way students learn due to the use of ICT tools. Knowledge of their learning styles, to personalize teaching, relying also on ICT tools, goes hand in hand with finding contemporary techniques and methods to develop students' key mathematical competencies.

The questionnaire made with mathematics teachers throughout Albania raised an immediate need for the training of teachers regarding the use of ICTs tools to be close to the learning modalities of each student to personalize the teaching of mathematics. Continuous training, concerning all the factors that guarantee success in their profession, is the key to success.

Teacher training for student learning styles and the use of ICTs tools to support teaching are key points for personalizing math teaching.

A key role in personalizing math learning is played by the teacher. The design of a personal learner profile (PPL) for each student, requires at the same time the identification of students' learning modalities. The teacher, on the other hand, must choose wisely the technologies that will support his teaching techniques to penetrate the students' learning styles effectively. So, identifying learning styles as well as choosing supportive technology are two very important components, which should be given priority as we seek to personalize math learning.

The students absorb the learning material in different ways, which are unique to each of them. Some of them use sketches, concept maps, graphics, and images to organize, process, and memorize. Others are active in debates and discussions, are creative, are attentive listeners, and take notes as they listen. Everything has to do with their learning style.

Learning styles are varied, they are in most cases found mixed in each student. This fact requires that the range of activities, strategies, techniques, and teaching methods be enhanced to support everyone according to their learning style or style.

It seems in fact that this adaptability is a multi-variable equation, as it is about a whole class, with individuals learning in different ways, at different rates, with different interests, and with different motivations.

But if we think about the supportive role that technology is increasingly playing in teaching math, this adaptation becomes somewhat easier.

Educational technology aids math teachers to support each student, according to his or her learning style or styles, as well as to gradually educate them in the multimodal way of learning, as the more multimodal the student is in the learning process, the higher their math scores are.

Today's students, growing up in the digital age we live in., are looking for math teachers who not only master the subject perfectly but are competent both pedagogically and digitally. The teachers need to combine two important elements of this process in the same classroom:

1. Efficient methods are carefully selected by the student's learning styles
2. Technological support enables those to progress according to their style, or learning styles.

1 LEARNING STYLES AND THEIR ROLE IN MATHEMATICS TEACHING

1.1 An overview of learning styles. Definitions and disegnation

Learning style is the complexity of factors, attitudes, and behaviours, which make it possible for a person to learn a certain material in the easiest way possible.

Regarding studies of learning styles, the most serious problem is the confusion abouttheir definitions. In the past two decades, the learning styles have been used in various and sometimes confusing ways in the literature. (UKEssays, 2018)

We can not give an opinion on which of the definitions of learning style is better or less good, because each of them has made the determination based on different perspectives.

Let's start with Keefe's definition (Keefe, 1979): The learning style is the composite of characteristic, cognitive, affective, and physiological traits that are relatively stable indicators of how learners perceive, interact with, and respond to the learning environment.

Let's look at some definitions first. The term **learning style** comes from general psychology. Rod Ellis defines learning style as *the characteristic ways in which individuals orientate to problem-solving*. (Ellis, 1994)

Learning styles are usually defined as the cognitive, emotional, and psychological traits of students when they interact in the classroom environment. Students with different styles try to solve problems in different ways. (Sternberg & Grigorenko, 1997)

Different authors when talking about learning styles, see them as ways through which students individually receive, process, and acquire diverse knowledge and information. Each style of learning has its perception, it can appear as (seeing, hearing, or touching) or it can be of the sensory type. Regardless of the content or type of student's learning style, mathematics should be conveyed to them in a way that is compelling and based on their strengths as, *we feel the need to emphasize that all humans, short of being afflicted with certain types of organic damage, are born with an astounding capacity to learn, both in the amount that can be learned in one domain and in the variety and range of what can be learned. Children, unless stifled in some way, are usually virtuosos as learners.* (Pashler, McDaniel, Rohrer, & Bjork, 2008).

Learning is fruitful if speaking, talking, are accompanied by actions, exercises and working, acting, etc. (Willingham, 2005)

Learning styles are essential for the successful teaching of mathematics. They are a powerful tool for selecting strategies, methods, techniques, and appropriate technological tools for successful teaching. The students do not learn in the same way. The use of different mental processes from the lower level to the higher level makes it possible for all students, regardless of their different learning styles, as well as the ability they have to learn, to progress.

This progress must be persistently sought because they have different abilities and each of them learns in different ways.

Modern schoolchildren are the new digital generation and their preferences for working with information are based on the dominant sensory modality which can be visual, auditory, and tactile/kinesthetic. Therefore, to organize effective mathematics teaching it is necessary to use a personalized system of teaching techniques, instructional methods, and educational technologies taking into account the way students process information from the outside world" (Sheromova, et al., 2020).

In the same study, they shed light on a neuropedagogical analysis of this issue, emphasizing: "In light of neuropedagogy achievements and the idea of dynamic balance, the purpose of this article is to provide evidence for the current need for taking into account the student's perceptual learning styles which reflect the nature of interhemispheric interaction in the process of cognitive learning development in mathematics learning" (Sheromova, et al., 2020).

A successful learning process requires that students be encouraged to learn. Students are known to learn with pleasure when concepts are served to them in a way that they find most appealing and in a way that makes it easier for them to assimilate these concepts more easily and quickly. This process requires detailed prior knowledge of the tendencies, needs, interests, and the way they learn better.

In short, it is very important to know the students' learning styles.

Learning styles also combine a series of internal factors that make it possible for students to interact with the environment by perceiving and reacting to the learning process as a whole, in different ways.

When discussing student learning modalities, we have to keep in mind that we should not talk about a student's learning **style** but about learning **styles**, as they are mixed regardless of the dominance that one can have over the other style, in this mix of them.

About learning styles, can not make definitions of the type "Good" or "Bad", they should be considered an asset to our students, while math teachers should identify them, adapt them to their strategies and methods of teaching, as well as to enrich the range of their learning styles so that regardless of the learning situation in mathematics their benefit will be optimal.

In the **pedagogical triangle** of **teachers**, **students**, and the **subject**, the approach of learning styles trains professionals to focus on how students learn or fail to learn. What is needed is equal attention to all parts of the triangle and their interactions. Here the risk is for us to end up with pedagogy without content, where the process is celebrated at the expense of content. For some learning style developers, there is no specific category of students with learning difficulties, there are only teachers who have not learned that their teaching style is appropriate, even in the same study they bring the example of the Dunn model (where students 'failure to learn is reformulated as teachers' failure to teach appropriately) which has transformed their attitude towards students who were previously dismissed as stupid, slow, unmotivated, lazy or uneducable. (Coffield, Ecclestone, Hall, & Moseley, 2004)

Mathematics teachers should strive to bring students closer to learning styles that they either do not have or have not adopted appropriately.

Let's bring here the example of solving an equation by two students with different learning styles (Figure 32&**Error! Reference source not found.**).

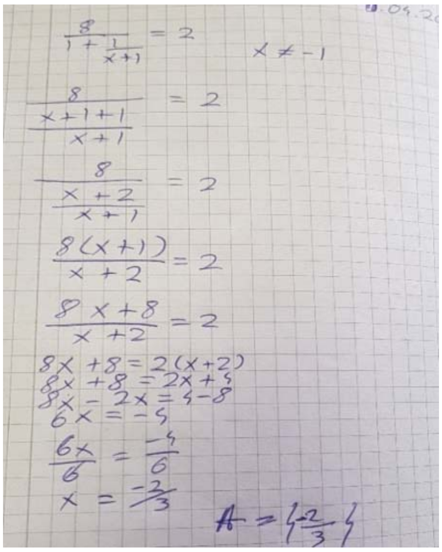


Figure 32. The solution is given by the students with the sensory style

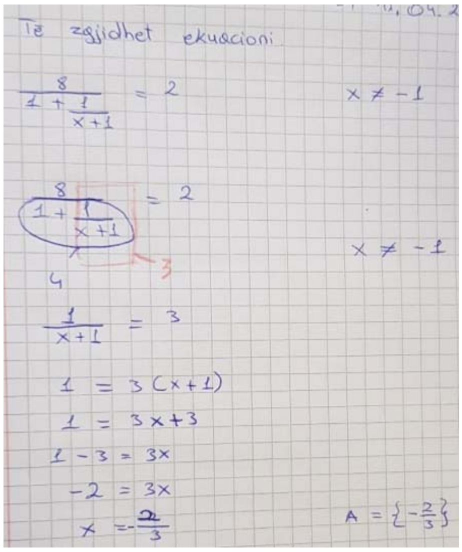


Figure 32 The solution is given by the students with the intuitive style

The sensory learner has solved the equation by respecting, one by one, the algorithms needed to solve an equation, while the intuitive learner's solution is brilliant. He estimated from the beginning the fact that the denominator of the rational fraction should be 4, and then solved a simpler equation, to find a value of *x*.

Intuitive students are often also gifted students in the classrooms. They always find a way to think differently and provide quick and accurate solutions. But if the solution offered by the intuitive learning style is often presented and elaborated in front of the class, intermediate-level students will certainly adopt it as a solution even though they do not fall into the category of gifted students in the class.

If such a student, with a sensory learning style, remains in his *habitat* without being drawn to intuitive reasoning, the role of the teacher as the instigator of his analytical and critical thinking has failed. But this process of attracting students to new learning styles requires first and foremost a very good understanding of their nature, strengths, and weaknesses.

Given the capacity of humans to learn, it seems especially important to keep all avenues, options, and aspirations open for our students, our children, and ourselves. Toward that end, we think the primary focus should be on identifying and introducing the experiences, activities, and challenges that enhance everybody's learning. (Pashler, McDaniel, Rohrer, & Bjork, 2008)

1.2 Learning styles. Classification and mathematical context.

Different researchers who have dealt extensively with learning styles have made quite different classifications of them, according to personal characteristics, behavioural traits, personal and environmental preferences, dimensions, etc...

Let's dwell on the most discussed learning style classifications below there are more than 4 learning styles and they are classified according to different criteria.

1.2.1 According to psychopedagogues and psychologists

According to **psychopedagogues and psychologists,** they are the 4 most prevalent learning styles.

1.2.1.1 Auditory learning style

Students who have this learning style remember better and have higher scores when listening. These students, even when reading, read aloud. Group discussions are the perfect *habitat* for them. In the group, they can discuss freely and benefit from what they hear from others.

Students who have this learning style often succeed in group activities, where they are asked to discuss learning materials orally with their classmates and they can benefit from reading their written work aloud. (Felder, 1988)

These students are the most attentive in the class and when it comes to illustrative videos displayed from a monitor on the SmartBoard they get the maximum benefit.

1.2.1.2 Visual learning style

Students who have this style of learning, learn through images, graphics, and figures. On the other hand, these students can perfectly present through sketches the deconstruction of a certain lesson. *Because visual learners tend to be holistic learners who process information better when presented to them as a whole rather than in pieces, they tend to see positive educational outcomes when presented with summary charts and diagrams rather than long text information.* (Stahl, 1999)

Students' notebooks have everything that is written on the blackboard or important notes that he has taken out during the study. Their pencil cases are filled with markers of different colours and their books have notes everywhere. *It is much easier to remember a vivid image as a photograph than toremember what someone said or wrote.* (Sternberg R. J., 1994)

So, to support students with this style of learning, the teacher should carefully process the mathematical content to present it schematically while keeping intact the essence of the material.

For visual learners, for example, instead of writing on the blackboard that the cuboid has 8 vertices, and 6 rectangular faces, which are 2 by 2 congruent, 12 edges, and 4 by 4 congruent, a cuboid can be used where to be coloured the same congruent faces and congruent edges. Another way is to choose the appropriate diagram to completely organize the content of the lesson.

1.2.1.3 Kinesthetic learning style

Students with this learning style will want to touch on everything they learn. We can say that these students seem to have undergone successive lessons of the Montessori method. The only requirement that would come from their subconscious towards the teachers of mathematics would be: *Teach me to learn myself, teach me to do it myself* (Age of Montessori, 2020)

This would be ideal if we take into account the role that the teacher has taken in active teaching. The learning style of those who prefer practice: they learn more by doing than by reading, watching, or listening. Kinesthetic students are the first to come up with the model of the project when they work on projects; when it comes to building the geometric body they have the imagination to sketch the opening of the body or to build it when only the paper is given. *Kinesthetic students tend to include all their senses equally in the learning process.* (Fahy, 2005)

In the new dimensions that the teaching of mathematics has taken, away from conventional classroom environments, based on the creation of mathematical competencies, learning environments must be created for them to engage as effectively as possible.

1.2.1.4 Learning verbal style (Literacy)

Students who have this learning style organize the whole learning process based on the textbook material. Their learning adheres to the routine: read - take notes - sketch - repeat - and read again. They read to understand, sketch to memorize, and then reread to fixate and learn. The examples solved in the book, and the examples solved in a class by friends or the teacher are for them the basis of learning mathematics. They are the students who tend to build maps of mathematical concepts and sketch the connections that concepts have with each other. This helps them organize their thinking, delve into the content and make the connection to real life. For students with this learning style, it is recommended to use active learning methodologies, mainly problem-learning and inquiry-based learning.

1.2.2 According to the sociopsychological or personal characteristics

According to other divisions, there arealso learning styles that have to do with **sociopsychological or personal characteristics** or are even influenced by the environment. These styles intertwine with the main learning styles by reinforcing their learning in mathematics.

Here are some of them and the features they display.

1.2.2.1 Active learning style

These students take an active part in the learning process. They involve their whole being in the learning process, use creativity, are impulsive, open people, and look forward to everything new with enthusiasm.

1.2.2.2 Theoretical style of learning

While we are looking for students who have a developed sense of logical, critical, and analytical thinking, we find them in students who have a theoretical learning style. The theoretical style reflects those who are methodical, critical, analytical, disciplined and use logic.

1.2.2.3 Reflective learning style

Who is better than a student with a reflective learning style, looking for some solutions to a certain problem and analyzing them from different perspectives? Their learning goes through the steps: **observation - analysis - meditation**. With the prudence that characterizes them, they are detailed and look into the depths of the problem.

1.2.2.4 Pragmatic learning style

Pragmatic students are characterized by objectivity. They are based on reality leaving aside abstraction. They are productive and give specific details in the answers provided to every asked question.

1.2.2.5 Sensory learning style

These types of students always tend towards practicality. They aim to provide solutions to mathematical problems by strictly following the steps for solving them, according to clearly defined procedures.For students with this learning style, the teacher needs to plan hands-on activities, and use active learning methodologies, project-based, problem-based, question-based, or nature-based excursions.

On the other hand, if there are children in the class who do not like to memorize facts that do not have a certain meaning, or to solve e.g. 20 equations of the same kind, which have the same solution procedure, these are the children who have an intuitive learning style.

They have a keen intuition to find an original solution that can be very short and very brilliant.

The classrooms are filled with students learning in different styles, having their original ways of learning, how they organize the solution of a particular exercise or problem, how they interpret it, or even how emotively they make this interpretation.

It often happens that it is a task or a mathematical activity that is entrusted to be performed by a group of students to be presented by one of them, who finds it difficult to present the results achieved by the group or by someone else who shows the material so beautifully that creates the idea he has done everything in that group task. We are dealing with traits that are essentially motivational, environmental, and emotional tendencies, in short with their affective traits.

Students' natures are varied, some of which are comfortable during group work, or when debating or discussing. Some timid students worry not only about the small noise of group work conversations but also about the presence of such a large group of students who work together on a project or to solve a problem. These students value solitude and feel comfortable workingand focusing optimally on their tasks when they have no one around.

1.2.2.6 Emotional preferences

Students' approaches to learning are of different natures, someone learns willingly, but someone else learns well under the pressure of the grade, the teacher, or under the pressure of the parent. It all has to do with motivation. Someone is self-motivated, someone else needs to be motivated. It is important to find the right way to reach our final goals regarding their learning. What we need to do in any case is to display the beauty and greatness of mathematics, to motivate students to move forward, and to create mathematical skills and competencies that are so necessary for everyday life.

1.2.2.7 Environmental preferences

A very important role, especially in the concentration to learn, is played by the environment. Many of the students' learning is closely connected to the learning environment, order, or the tranquillity it offers, although others are very focused on what they will do and despite the unfavourable environmental conditions, they achieve the same result.

1.2.2.8 Competitive

It is about those students who want to be the protagonists of the class, who win every competition, want to be the first to solve a task, and always be in charge.

1.2.2.9 Co-operative

For this category of students, every mathematical concept that is taken is interesting. They see learning as an exchange of ideas, a constructive debate. For them, learning means sharing ideas and interacting.

1.2.3 According to the level of dependency

1.2.3.1 Dependent style

But in class, often have someone who is always waiting for you to show them the steps to solve a problem, they follow step by step all the instructions. And the whole process of teaching them ends there, with the strict implementation of the instructions given. Students with this learning style belong to the **dependent style**.

1.2.3.2 Independent style

Students who have an **independent style** as learning style are the ones who force us to search the Internet for a Website that has material related to the learning process, to guide them to navigate there, with maximum curiosity and the highest motivation.

The role of the math teacher is to see students learning abilities and styles not as static quantities, but as quantities that tend to develop and expand.

There are many variables to the ingredients for a good mathematics lesson, but most importantly, know what and how you are teaching, provide opportunities for all students to achieve success, and be enthusiastic and passionate about mathematics! (CATTARD2017, 2018)

Learning mathematics is a multidimensional process that aims to create skills and competencies and not just to acquire knowledge.

One of the core values of teachers is support for all students, regardless of their learning style, strengths, or weaknesses. Promoting the academic, practical, social and emotional learning of all students is an important competence for this value. The in-depth competencies related to what is defined in the role of the inclusive teacher are (Unicef Albania, n.d.):

- Know the ways students learn/absorb and the teaching models that support the learning process by focusing on identifying a student's strengths.
- Differentiation of the curricular content, learning process, and didactic materials, to involve students and meet different needs.
- Personalized learning methodologies for all students that support and promote their learning autonomy.
- Use of equipment and aids in support of the learning process;
- Planning teaching and learning based on the needs, interests, and ways in which the student perceives the environment. Combining tasks and requirements for students with their skills and abilities.

- Identifying the student's strengths and then combining them with weaknesses;
- Promoting collaborative learning, where students help each other in different ways within flexible groupings of students;
- Systematic use of a range of didactic methods and tactics;
- Use of communication technology in support of flexible learning techniques;
- Use of evidence-based teaching methods to achieve learning goals, alternative learning paths, and flexible teaching;
- Use of formative and summative assessment that supports learning and does not label or lead to negative consequences for students.

2 ICTSANDLEARNINGSTYLES INMATHEMATICS TEACHING/LEARNING

Digital technology plays an important role in the development of many students' abilities ranging from those related to critical and logical thinking to skills related to their cognitive and emotional development. With the endless range of tools that ICT offers, as well as the various software platforms designed to teach mathematics, the latter is always closer to every student, regardless of the strengths or weaknesses,he has, regardless of how he approaches mathematics learning, regardless of the interest he has in learning mathematics, or the way he studies it.

Digital educational technologiesprovide also a valuable asset for teachers, as through it they have the opportunity to reach out to students with mathematical knowledge in ways that are very appealing to them and that make mathematics learning as interesting and productive as possible. Let's see how technology reinforces and supports students with these three learning styles:

1) visual; 2) auditory; and 3) kinesthetic.

2.1 Learning by visual style and the role of ICTs

Educational ICTs tools enable students with the visual learning style to feel comfortable while surfing the internet, on math education pages, in math games, in instructional videos, where concepts are explained, or where they can practice to acquire mathematical skills and competencies.

Various demonstrations of graph construction or the study of linear functions through GeoGebra help visual learners to understand and memorize in a way that for them is one of the strengths of mathematical learning.

Preparing materials in PowerPoint or Prezzi, where they are presented with colour illustrations and animated effects, elements of different geometric 2D shapes (vertices, angles, diagonals, etc.), and graphic illustrations of the characteristics of different figures would be finding the right one for students with this learning style. But even if the insufficient technological infrastructure of the school does not allow such a thing, the teacher should first find videos online on various platforms or web pages, and recommend students to watch them beyond the time designated to teaching mathematics in the classroom. Although these videos may be short, they are effective in understanding a concept or practising it.

A very good way would be e.g. orientation towards the Akademi.al (https://www.akademi.al/) platform, offered in Albanian, where students have an explanation of the mathematical concept accompanied by illustrative videos.

For the little ones, (grades 5-6) the math games that enable them to practice while playing and having fun with their mobile phones play a very fruitful role in memorizing facts about mathematical operations, or more broadly about the properties of 2D shapes, geometric transformations,etc.

(Example:EgLearnMath&Mathproblems.https://play.google.com/store/apps/details?id=com.companyname.MaturaMatematyka)

All the visual aids that the teacher can prepare for a certain learning unit, not only provide a comfortable lesson for the students of this style but also serve as supplementary material for the forthcoming students as well, gradually creating in this way an appropriate database with digital didactic material that will help teachers in their daily work.

2.2 Learning by listening (auditory style) and the role of ICTs tools

Speaking and listening are the two ideal ways in which information reaches students with an auditory learning style. Technology offers golden opportunities for this category of students. They must be carefully selected to have the right effect. Good practice was observed in Albania at the beginning of the Covid-19 pandemic, where teachers were forced to switch to online learning, at a time when let's say that their knowledge in the use of applications related to teaching mathematics left to be desired. This moment showed that if something is useful, it is necessary. So the teachers recorded their explanations for the math subject mainly in **Loom recording**(https://www.loom.com/screen-recorder) which is one of the bests screen recorders from Chrome, Mac, Windows, iOS, and Android. It is a technological tool, easy to install and use, and free.

Explained and recorded teaching materials were then sent to the students, first in WhatsApp groups, which were created immediately after the schools closed, and then in the classrooms that opened in the Google classroom. Over time they began to schedule meetings there to conduct the full lesson, where through the **Jamboard**(https://jamboard.google.com/)they built schemes or figures in advance, concerning the material they had to explain and then allowed students from their homes write on the screen as if they were in class.

This period proved that without technology we would have done almost nothing for teaching and learning.

2.3 Kinesthetic style learning and the impact of ICTs tools on this type of learning

The basis of this learning style is practical projects and active learning. New information comes very naturally to them if presented as an interactive game.

Applications with mathematical content can help students while learning on their tablets to learn mathematics and develop logical, critical, and computational thinking. For students with this learning style, there are endless math sites online that adapt this learning style even by playing interactive games and learning new concepts. Through the computer, the teacher can simulate learning situations to urge their imagination towards the connections that the lesson we have to explain has with real life.

Through websites that offer free interactive simulation programs, students can develop their skills and competencies in the field of mathematics as well as learn according to their learning style. The most widely used interactive math content online games are a great help for students with this learning style.

The integration of technology into teaching mathematics based on the many technical support tools in circulation offers many opportunities for students of all learning styles. It is in the interest of students and teaching that most students have several styles of learning intertwined at the same time. The teacher can use a variety of carefully selected technological tools thus reaching all students, regardless of the student's learning styles.

Let's mention e.g. explanatory illustrations provided by **KhanAcademy** (https://www.khanacademy.org/) to reinforce a concept or to be tested on a set of topics and knowledge. Within it each student finds material to practice based on his or her strengths, uses strengths to improve weaknesses, walks at his or her own pace, does not need to be in the classroom or a formal educational setting, and in this way he removes the barriers to self-direction by meeting his individual needs for a quality mathematical reading. Through technological tools and computer software, the range of active teaching strategies and techniques increases, orienting the student towards lifelong learning.

3 TEACHING STRATEGIES, LEARNING STYLES, AND THEIR AMPLIFICATION THROUGH ICTS TO PERSONALIZE THE TEACHING OF MATHEMATICS.

Combining the two components: teaching strategies on the one hand and educational technology, on the other to be close to all types of student learning styles, is a task that requires commitment, integration of technology with lesson plans, other sources of teaching, and above all requires perfect planning for all stages of the lesson as well as the rigorous implementation of the planning done. In this equation with **4 variables, – Personalization of mathematics learning**, is expressed depending on the **other three**, which are: **teachingstrategies**, **learning styles**, and **application ofdigitaleducational technology in learning mathematics**.

Each of the other three variables on which personalized learning in mathematics depends is influenced by a series of factors, of different natures that can be of a psycho-pedagogical or motivational nature. Some tips that a math teacher should consider when looking to personalize math learning are:

1. Give space to the mathematical intuition and fantasy of each student, building the teaching of mathematics on the active methodologies of learning. Create comfortable spaces for them to *learn by doing* (Dewey, 1974).
2. When it comes to projects presented by students or problems for solutions that you serve, make sure that the situations presented are such that they require their decision-making.
3. Encourage students to build curriculum knowledge and elements by applying everything they know at the same time, solving real-life problems, and concluding on important results.
4. Encourage students to work on projects taking advantage of technology to produce collaborative and high-quality mathematical products, focusing the entire learning process on the student rather than the curriculum. This focus is influenced by the global world and brings tremendous benefits such as motivation to learn, passion for doing and flexibility.
5. Encourage students, while standing by them as a *critical* eye. Encourage them when they are on the right track, but rather encourage them when they are failing so that they can try again and again until they succeed. Their motivation is the basis of success in life.

6. Try not to exert any kind of influence or pressure, especially on the time they have available.
7. Rely on their strengths and learning styles, to encourage the improvement of their weaknesses and increase the variety of their learning styles.
8. Seek the support of technology at every stage of the process, seeing it as an amplifier that can help wherever students are and whatever they are doing.
9. Try to re-dimension your role, always being by the side of qualitative changes in mathematics education, based on active methodology and technological support.

One possible presentation of this complex process could be as in Figure 33.

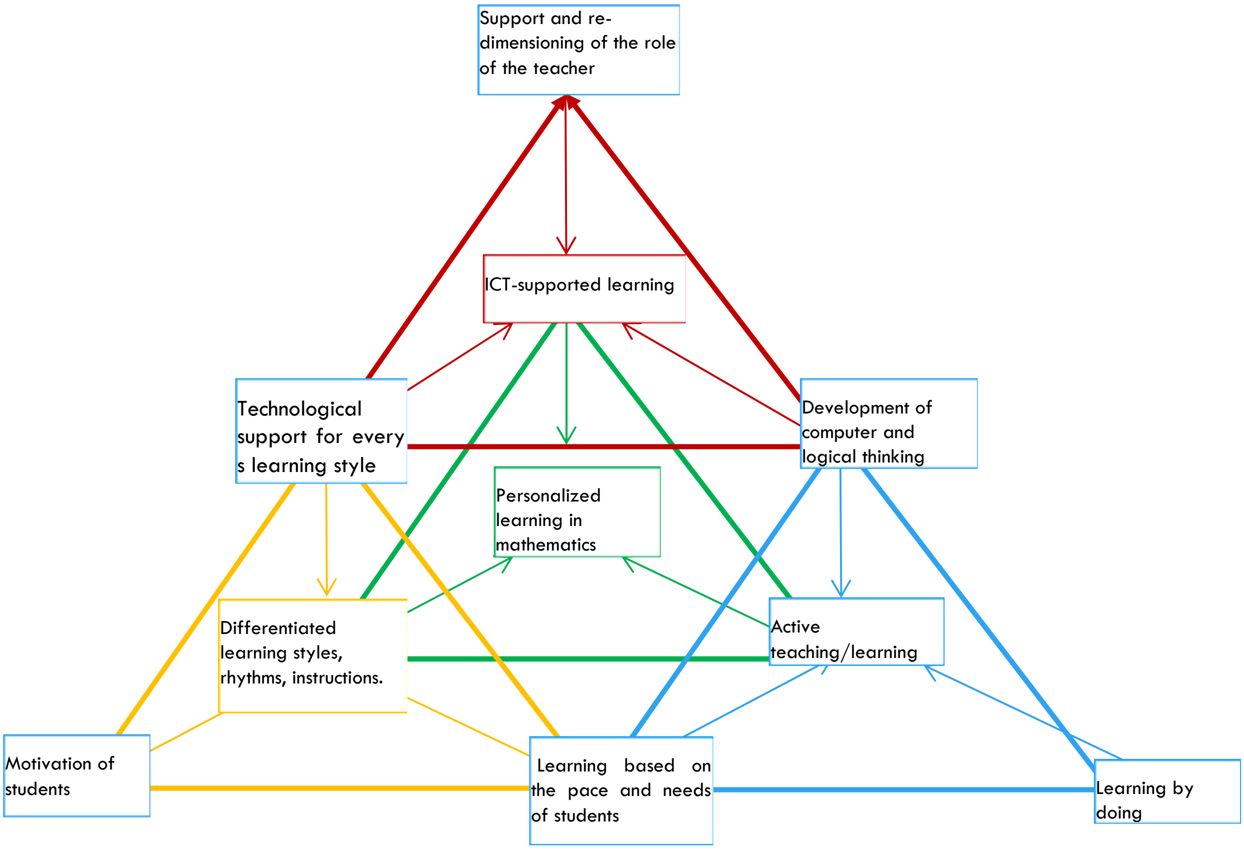


Figure 33Personalized learning and connection with other components.("Pyramid of personalized learning")

4 MATHEMATICS TEACHER'STRAINING ON MEETING THE PERSONALIZED TEACHING REQUIREMENTS

4.1 Teachers' training – the key to building a solid learning foundation in math

For the development of students' key mathematical competencies, as one of the narrowest goals, as well as for the improvement of the student's personality in all its dimensions, through educational activities with a global character, the continuous training of teachers is necessary. Itcould change the teaching from the traditional one to the student-centred one, where the teacher also knows the different characteristics of each child, knows his ways and styles of learning, knows his strengths and weaknesses, and offers each of them personalized learning in educational technologies.

For this reason, training that combines teaching and learning and ICTs tools as support should be the priority of training agencies.

According to the current laws in Albania, teachers must be compulsorily trained for 3 days, i.e. 18 hours which is equal to1 credit.

There are 54 training agencies for teachers located in 12 cities. The structures that carry out the training in Albania are:

- Ministry of Education and Sports;
- Institute of Educational Development;
- Regional Education Directorates;
- Local Education Offices;
- Schools;
- Universities;
- Non-profit organizations.

The categories in which the training takes place are very diverse. They include:

- Ethics, communication, pedagogy;
- Cross-curricular topics;
- Developing TEAchers competencies for a comprehensive VET System in Albania – TEAVET;
- Curriculum planning;
- School management and professional development of teachers;
- ICT in teaching and learning;
- Teaching and learning;
- TEAVET- PILOT(https://trajnime.arsimi.rash.al/)

The role of the teacher remains decisive when thought of as a person who not only imparts knowledge but takes care to develop the skills and competencies of a lifelong learner.

Rigorous implementation of innovations in subject curricula in general and in mathematics, in particular, requires effective training of teachers, on all issues related to the modern student-centred teaching process and which prepares competent generations, to cope with successful life challenges.

One of the main points to be reflected directly from the analysis of the questionnaire developed with the mathematics teachers was the low number of training they had received regarding the use of ICTs tools in teaching, by the education authorities during the last 5 years. 80% of them had received at most 3 pieces of training. (16% had no training, 24% had received only 1 training, 23% had received 2 pieces of training and 17% had received 3 pieces of training). This number of training is very low, considering the pace at which educational technology is developing.

4.1.1 Purpose of the training

The purpose of the trainingconducted with the teachers of mathematics in the region of Elbasan as well as the teachers of mathematics of the non-public school "Vinçens Prendushi" in Durrës (part of the network of Catholic schools in Albania) was to provide a model for teacher training based on aspects of teaching student-centred learning styles, supported by educational technology to personalize the teaching of mathematics.

4.1.2 *Training objectives*

At the end of the training teachers should be able to:

1. Recognize students' learning styles as a powerful tool for lesson planning and finding effective methods to develop subject competencies.
2. To adapt the efficient technological educational tool, based on the learning styles of the students to personalize the teaching of mathematics.
3. Use SmartBoard, the GeoGebra app, and Kahoot! App to support teaching and students'personalized learning in mathematics.

4.1.3 *Duration of training*

The training took place from 20 to 21 March 2021, in presence, in the premises of the school "Imelda Lambertin", which has a very good technological base and halls with large capacities. According to an agenda approved by the chairwoman of the professional network of mathematics teachers in Elbasan.

4.1.4 *Topics of training*

The main topics of the training were:

1. Learning styles in general and in mathematics in particular.
2. Technology and learning styles.
3. Personalized learning in mathematics based on the learning styles and technology support.
4. Use of SmartBoard, GeoGebra, and Kahoot! to support personalized teaching and learning in mathematics.

4.1.5 *Methodology*

Teacher training followed these steps:

Step one: The group of math teachers who teach in secondary schools in Elbasan, underwent mixed structured and unstructured interviews, to gather their opinions about students' learning styles, and their identification, taking them into account in finding supportive methods and techniques to realize personalized teaching. The interviews also focused on gathering information on the educational technology platforms and software they used most in teaching mathematics.

Step two: Based on their opinions, **the training topics were determined** and the training material was prepared.

Step three: Carry out the training in two consecutive days by rigorously implementing the program and schedule set in the preliminary agenda signed by the head of the professional network of teachers.

Step four: realization of a questionnaire on the effectiveness of the training as well as determination of the schools where the experiments will be implemented.In the two-day training that was conducted with the mathematics teachers of the Elbasan district, after identifying the characteristics of the students 'learning styles, the priorities of each of the types of students' learning styles were addressed, to make them successful in mathematics and brought facts about the opportunity that technology offers in the formation of mathematical skills and competencies, based on their learning styles.

4.2 Teachers' preliminary attitude to personalized teaching and the role of ICTs for its ensuring

In the training conducted with the mathematics teachers of the Elbasan district, on learning styles and the support of ICTs tools for the personalization of teaching and learning mathematics, it was noticed that mathematics teachers are convinced that technology has changed the role of the teacher. They know best that although educational technology can provide conveniences and adapts very well to students' learning styles, it will never be able to create the motivational leadership and human relationships that a teacher creates with his students. Some of them think that knowing the learning styles of the students is not of great importance as the variety of strategies and methods they choose to make the teaching of mathematics attractive encompasses all learning styles.

Teachers'knowledge of learning styles and educational technical support based on students' learning styles at the beginning of the training is presented in Figure 34.

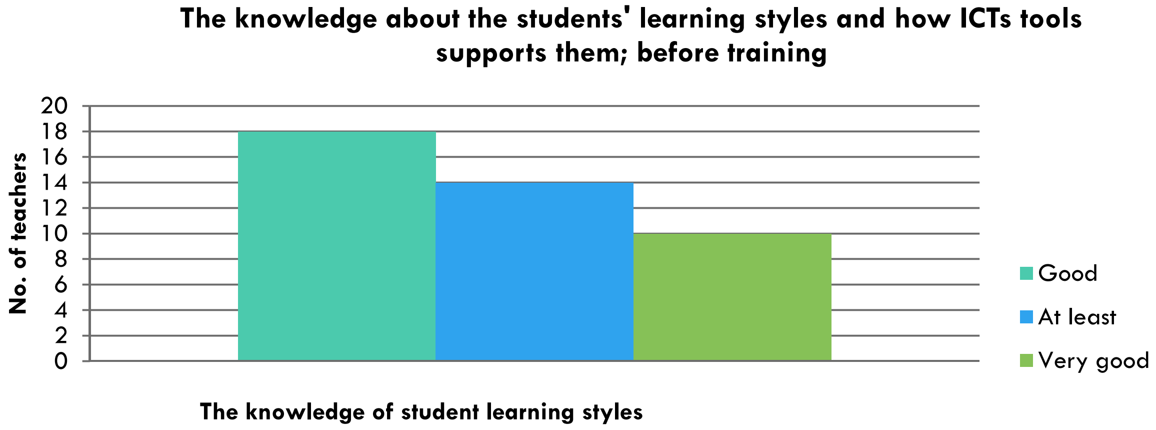


Figure 34 Knowledge on students' learning style and the support of ICTs tools

Mathematics teachers have general knowledge of learning styles, as during their training to become teachers they gain knowledge not only of their scientific formation but also of psychopedagogy. But to what extent these learning styles are supported by the use of technology is a very important topic that requires updating, as educational technology is changing very fast. The situation was more or less similar in terms of prior identification of learning styles for students in the classrooms where they taught.If teachers knew their students' learning styles they could more easily personalize the teaching, as they would be able to explain different mathematical notions and concepts more understandably and would also know how to carefully choose the platform, software, or simply the technological tool they would use to personalize the mathematics teaching. *If a teacher can identify trends in student learning styles, this will be very useful for developing teaching and learning processes for the convenience of student learning.* (Sujadi, Arigiyati, Utami, & Kusumaningrum, 2019)

In the opinion of the mathematics teachers who participated in the training, the identification of learning styles for about 40% of them does not matter, for 32% of them it matters, for 14% of them it does not matter much because the variety of the strategies, techniques, and methods they use in teaching normally that is effective for students of any learning style. Only 14% of them think that it is very important to know the learning modalities to adapt to their needs during the personalization of mathematics learning (Figure 35).

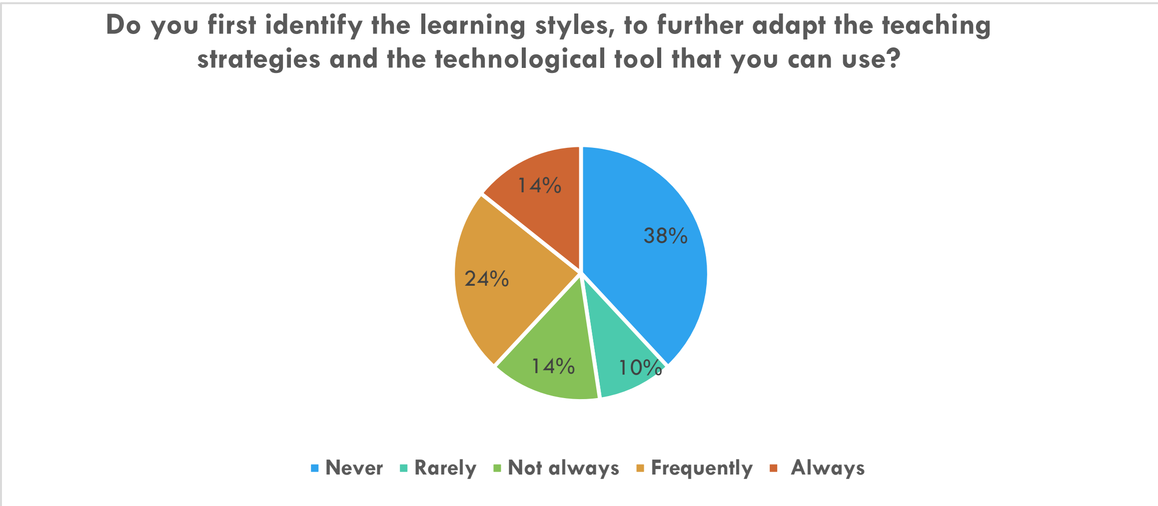


Figure 35. Importance of identification of the students learning styles

To personalize mathematics teaching, it is necessary not only to identify the learning modalities, the need, and abilities but also to create a personal profile of the learner (PPL) of each student so that the process of personalization of learning mathematics can be essential and not a sporadic random process.

Regarding the question: "*How often do you build a PPL in advance for the students in the classrooms where you teach, to personalize the math learning?*", the following results were obtained(Figure 36).

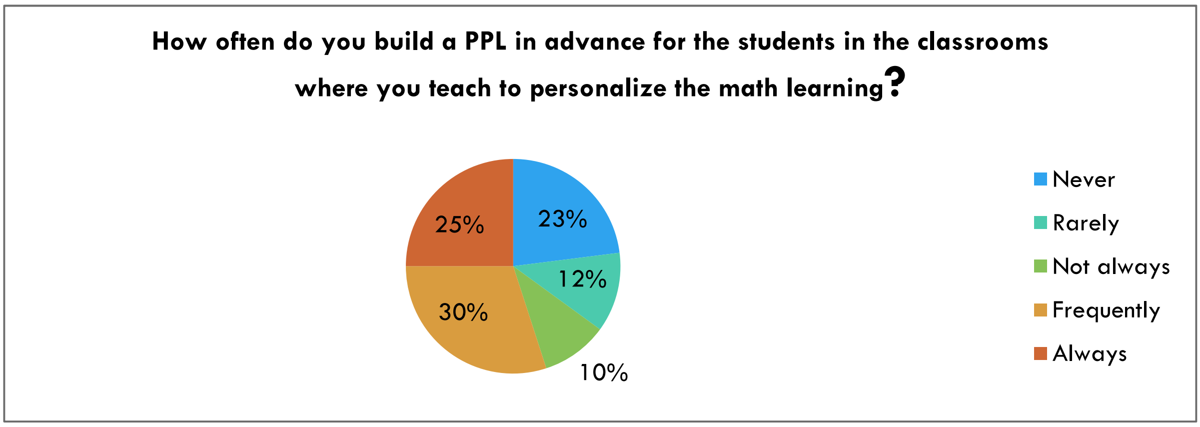


Figure 36 Periodicity of building a PPL in advance for the students

While 67% of them build such a profile for the students whom they teach, only 30% of the latter always refer to it as a guide in their work. Keeping track of each student's progress helps not only to guide them in their daily work but also to keep track of their progress.Meanwhile, a bigger part of math teachers considersa large number of students in the classrooms a hindrance to teaching each student according to his / her learning style and pace.

About the question:"*Can you, during your work, individualize math learning, thus teaching each student according to the style that he/she feels more comfortable with and more productive?*",the following answers were received(Figure 37):

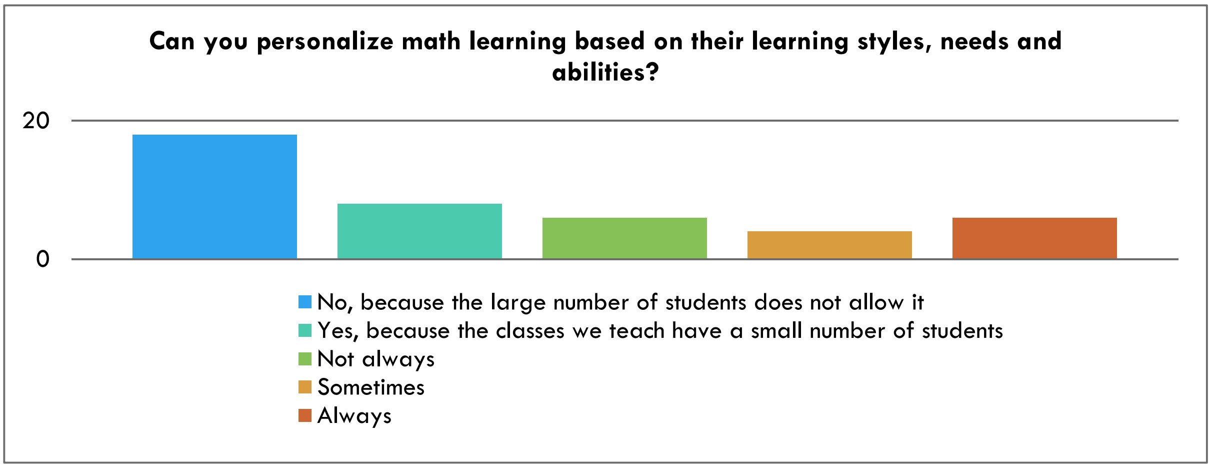


Figure 37Tendency to apply personalized learning

4.3 Teachers'training delivery

4.3.1 Teachers' training characteristics

The training on learning styles and the support that educational technology offers, to personalize the teaching of mathematics was a direct training, which was held on 20-21 March 2021, and was organized in the premises of the school "Imelda Lambertini" in Elbasan. The training developed had the following characteristics:

1. Type of training – direct.
2. Title of the training program: Professional development of mathematics teachers.
3. The theme of the module: Identification and use of learning styles, supported by ICTs tools, to provide personalized mathematical education.
4. The number of hours of the training module: 18 hours (9 hours of direct training + 9 hours of reflections and group discussions.
5. The activities were defined in advance as in the agenda (Appendix 5).
6. Beneficiaries of the training: mathematics teachers of secondary schools in the region of Elbasan, as well as mathematics teachers of the "Vinçens Prendushi" Durrës school.
7. Expected results:
    a. To develop appropriate skills to Identify learning styles in the classrooms where they work based on the test model discussed in the training (Appendix 3), as well as the theoretical basis treated there.

b. To develop the ability to design a PPL for students as a reference model for personalizing mathematics teaching.
c. To develop digital competencies to adapt the learning style to the technological educational tool, to personalize teaching.
d. To develop competencies to use SmartBoard, GeoGebra, and Kahoot! to personalize the teaching of mathematics.

4.3.2 *The teachers' training process*

The training used direct training based on group work, discussions, and reflections.

On the first day, the first 15 minutes were devoted to their presentation and they were asked to express in one word the connection they have with technology in teaching.

Another 15 minutes were devoted to collecting their opinions on the expectations of the training, if they have received other training of this type, how effective they have been, etc.

Their recorded expectations served as a basis for comparisons with their opinions on the evaluation of the training, expressed in the final questionnaire.

Gathering information on whether they have received other training of this kind, in other training was aimed at measuring their interest in positive discipline.

The training took place after a general opinion of mathematics teachers was obtained in the national survey on the use of ICT in teaching mathematics, as well as interviews conducted with various mathematics teachers in the district of Elbasan. So there was good information on their digital competencies and the need they have for this training.

For the presentation of the theoretical material that will present the basis of group work, a PowerPoint presentation is used.

The topics for discussion and reflection were fully defined according to the agenda in (Appendix 5).

The second day of training had more practical character. Some photos of training days 1 and 2 are in Figure 38below:

Figure 38 Photos of the training days

The teachers after receiving the instructions on the use of the platforms that were discussed were divided into groups of 6 teachers in 7 classes of the school and implemented some of the instructions received in the training. Their work was monitored to see the progress of the implementation of digital knowledge acquired, as well as to intervene in case of uncertainty.

In the end, the trainees were subjected to a questionnaire that collected data on the validity of the training, as well as the challenges and difficulties they encounter in the process of personalizing the teaching of mathematics through technology.

At the end of the training, three teachers voluntarily agreed to experiment in their school with the knowledge gained in the training, to personalize the teaching of mathematics.

Mathematics teachers were enthusiastic about the type of training, as this training according to them met their professional needs in three areas:

1. Learning styles
2. Personalized learning
3. The concrete platform on how teaching can be personalized through them helps students of any type of learning style.

4.4 Teachers'training results

The interest of mathematics teachers in the use of technology in teaching mathematics is growing in value. The Covid-19 pandemic period necessitated the use of educational technology in the teaching ofmathematics. So, for many math teachers, the support tools and platforms mentioned in the training were familiar.

The novelty of the training was how they can be used based on the learning styles of students in their classrooms. Regarding the question: "Have you used any of the platforms discussed in this or other training to reinforce the teaching strategy in line with the student's learning style?",90% of them had used the platforms discussed in the training or similar platforms, but 60% of the latter, which means 56% of all teachers, said that although they had used them, they had not chosen them based on the learning style of the students but were inclined by the fact that the range of tools they provided, ensured good technological support. Almost all the teachers participating in the training agreed that their students would increase their performance in mathematics, based on their skills and needs, if they would use technology to support their learners based on their styles of learning.

Of course, technology plays an indisputable role in improving students' mathematical performance, but its benefits are multiplied when supported by a detailed *acquaintance* of each student in social terms, pedagogy, in their intellectual abilities, in their learning styles, in everything that has to do with the child's's as a person in the first place and then his abilities and needs.

This is why teacher training on each of the aspects that lead to the deep understanding of students, with the mission of developing their thinking, is a very important component that makes possible the development of technology on par with learning.

According to Kolb, *individual learning styles are complex and cannot be easily reduced to simple typologies.* (Kolb, 1984)

Learning styles, and their connection to technology is a very hot topic of education in general and in mathematics in particular.

As early as 2009, Collins (Collins, 2009)stated that Education today must meet the needs of students who are more comfortable in electronic environments, as well as those who need a four-wall classroom.

The ability to use learning style research to accomplish both will lead to improved student learning and a more productive experience.

The situation is almost the same today. The dilemma lies in the complexity of learning math, not so much in terms of the difficulty the subject carries, but in finding the optimal solutions that bring together different learning styles as well as the growing range of technologies supporting each. (Collins, 2009)

In the questionnaire conducted in the last part of the training, participants shared their opinion about the effectiveness and expectations of the training.

Thus to the question *Evaluate from 1 (I did not benefit at all) – 5 (I benefited a lot) the effectiveness of this training about learning styles*, the participants gave answers according to the graph (Figure 39).

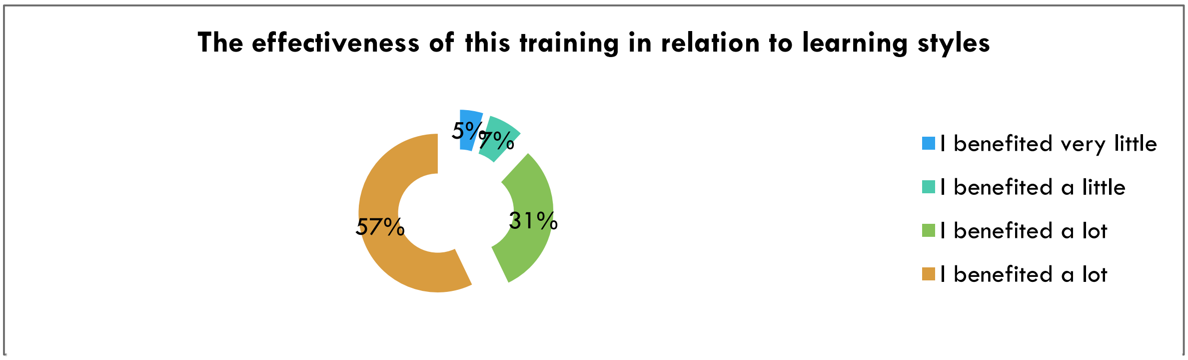


Figure 39. The training effectiveness in terms of learning styles

Regarding the question *Evaluate from 1 (I did not benefit at all) – 5(I benefited a lot) the effectiveness of this training about the platforms and technological mathematical tools that were discussed were given the following answers* (Figure 40):

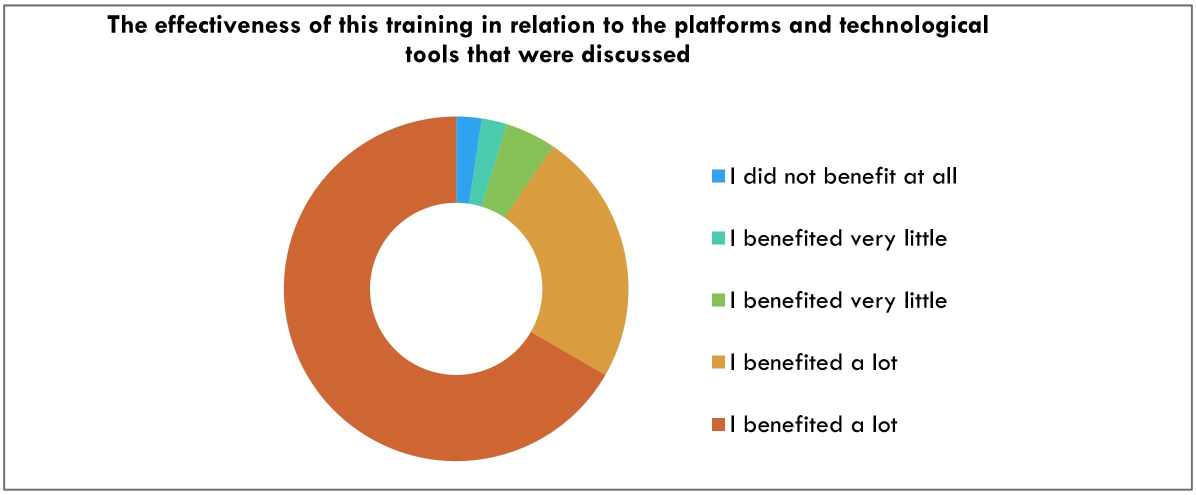


Figure 40. The training effectiveness in terms of using ICTs in math teaching

5 CONCLUSIONS AND RECOMMENDATIONS

All human beings are subject to a continuous process of learning. Life itself, in its entirety, is a learning process. In this learning process, we perfect the innate ability to learn by adding both the range of ways in which we learn and the skills and competencies we acquire. Our students come to life with an amazing potential for learning, a potential that must be pursued, cured, and expanded with extraordinary care.

They possess the gifts and talents that *sleep* within them, and they wait for someone to carefully *awaken* them, to take care of them to grow and to be gradually perfected. Teachers have this delicate and multidimensional mission.

The future of society as a whole depends on the way teachers are formed, on their coherence with all the changes that occur in their professional aspect as well as on the technological developments that enable their professional growth.

They must undergo continuous training, to be updated with both technological developments in the field of education, as well as innovations in teaching while staying in coherence with global developments in all areas of society.

Although teachers need to know their students' learning styles to adapt them to their teaching strategies, incorporating as many techniques and methods as possible into mathematics teaching can enhance their intellectual abilities, although sometimes the strategy used in teaching may not suit the learning style of any of the students. The variety of techniques and methods used in teaching mathematics, educates every student, gradually a variety of learning styles, although they may not be as powerful as the main style of learning.

The training developed with mathematics teachers showed that, despite the work experience of mathematics teachers, despite the high level of science they have in mastering the subject, they must constantly apply teaching methods that combine psycho-pedagogical and methodological aspects with the support of ICTs tools. Student success in mathematics will only be achieved if teachers carefully combine the following four components:

1. Personalized learning;
2. ICT support;
3. Adapting teaching strategies to students' learning styles;
4. Optimal management of all stages of the learning process.

In Albania, which is a developing country, mathematics can play an important role in transforming the whole society. For this reason, continuous training of teachers regarding mathematical content, pedagogy, and active teaching methodologies should be a priority of educational policymakers.

Recommendations to the policy-makers:

1. Possession of digital competencies for the application of educational technologies in the teaching process should be an obligation for teachers and to be controlled and followed with transparency by the authorities. It is recommended that the qualification tests that take place after (5, 10, and 20 years of work) also include questions that test the digital skills of teachers.
2. Monitoring of Teacher Training Agencies, for a more equal distribution of the developed training, giving the training related to ICT tools in education the appropriate space or reorganizing the training agencies by profiling them to offer training in 1 - 2 categories.

3. Monitoring of the training they perform after the training plan has been approved by the competent bodies, bringing under control a fair distribution of training according to the fields.
4. Establishment of teacher mentoring structures by qualified teachers towards the use of ICTs tools in teaching, motivating the latter for the service provided.

CHAPTER 5. VALIDATION OF TEACHERS' TRAINING ON PERSONALIZED MATH TEACHING, SUPPORTED BY ICTS

Technology alone isn't going to improve student achievement. The best combination is great teachers working with technology to engage students in the pursuit of the learning they need

Arne Duncan

Teachers undergo periodic training throughout their teaching career. Through them, teachers update their knowledge about contemporary methodological treatments of the teaching material as well as about the use of educational technology in the teaching of mathematics.

The training of teachers conducted in Elbasan, regarding personalized teaching in mathematics, based on learning styles and supported by ICTs tools, ended by introducing teachers to the results of an experiment conducted at the school "Imelda Lambertini", on the support brought by the use of SmartBoard for the personalized teaching/learning of mathematics, in parallel with a detailed planning and a deep study of the abilities, needs and talents of the students made in advance by the teacher. Immediately after that, three teachers from different schools volunteered to implement the **first experiment** "Personalized teaching and learning mathematics through problem-solving, supported by SmartBoard" in their schools.

Due to the size of the sample, a preliminary study on the quality of ICT educational tools in the school "Vincens Prendushi" in the city of Durrës, as well as the digital skills of the teacher during the training, this school was chosen to implement the experiment. The experiment lasted 8 weeks (34 teaching hours of which 32 teaching hours and 60 minutes for the quarterly test).

The study population was seventh-grade students, while 79 seventh-grade students from this school (experimental group) and 60 students (control group) from school "14 Nentori" in the city of Durrës underwent a controlled experiment.

The second experiment on "The efficiency of dynamic GeoGebra software, in giving differentiated instructions based on student levels, on treatment concept of functions in the eighth grade", was decided to be conducted at the school "Jorgji Dilo" in the city of Elbasan. This experiment was also conducted in the third quarter of the school year 2020 - 2021.

The selection criteria were the sample size, the quality of technological tools in the school, as well as the digital skills that the teacher displayed during the training. The experiment lasted 8 teaching hours and the study population was the eighth-grade students with sampling, 3 eighth grades of this school (experimental group) and 3 eighth grades of the school "Qamil Guranjaku" in the city of Elbasan (control group).

The third experiment was about "The impact of learning games on the approach of secondary students towards the subject of mathematics and the development of their computational thinking". lasted 8 weeks and the students taken in the study were the students of 5 classes (Grade 5, 6,7,8,9) with a total of 150 students of the secondary school "Imelda Lambertini" in Elbasan.

Let's deal with each of them extensively.

1 PERSONALIZED TEACHING AND LEARNING MATHEMATICS THROUGH PROBLEM-SOLVING, SUPPORTED BY SMARTBOARD

1.1 Introduction

Technology is facilitating our activities in all areas of life. With the comfort it offers us, it brings us close to many services that a few years ago were not imagined.

Fortunately, many learning activities have been facilitated and technological tools have been added to support education. In the variety of technological tools that support an innovative teaching process, a SmartBoard would be an innovative educational "package" that offers at the same time, information, convenience, interaction between students, and access to many different applications that support the most learning.

Implementing a SmartBoard in the classroom moves the teacher from the "comfort zone" to an area where he or she may not initially feel so comfortable "digitally" due to not-so-good knowledge in the field of technology.

By bridging this gap, the teacher begins to realize that this technological tool transforms his teaching, making learning a process that aligns with students' desires to engage with technology.

The fact that a math teacher needs very few days of training to learn how a SmartBoard works and how to use it and all the benefits of using it, makes it immediately very approachable and enjoyable, integrating diverse effective technologies within a lesson. Moreover, it provides them with comfort in use, away from the chalk, and wipers, as well as saves the time of presenting the material, solving an exercise or a problem, etc.

Smart Boards create opportunities for teachers to support each student, according to his / her learning style, in a personalized way, to develop mathematical competencies.

It offers a variety of visual aids, including geometric figures, sketches, and graphs marked in different colours, which are used to illustrate mathematical concepts and knowledge that will be constructed in a given hour. Students are offered interactive explanatory videos, and learning games accompanied by relevant explanations, which bring the teaching material more comfortable and attractive, with the style of listening learning and more.

Nowadays, where all students are attracted by ICTs tools, not only students who have the auditory style of learning but all students, who expect that their mathematics learning is built interestingly and easily acquired.

Also, kinesthetic students have endless fun with the SmartBoard touch screen, becoming curious and at the same time interested in everything related to its use.

The teacher can prepare the teaching material at home and bring it to the classroom via e-mail. Once the lesson is over the lesson material can be saved to support missing students' learning or to create a database for the following years.

One of the students may want to revisit the notes or sketches made to illustrate a certain problem, as his learning rhythm is not like that of others. He has the opportunity to go back to class after school, open the SmartBoard and review the material stored there.

These are the benefits that can not be obtained from the use of blackboard and chalk. The Covid-19 pandemic period once again reinforced the role of SmartBoards in teaching in general and in mathematics in particular.

1.2 A brief overview of the benefits of SmartBoard in teaching mathematics.

It took 190 years to move from the use of the blackboard (1800) in teaching to the advent of the SmartBoard in 1990.

The year 1960 was a transitional period where the transition was made from blackboard to whiteboard (1960) and then technological grit, so brought the use of interactive whiteboards. The first to invent the interactive whiteboard was the Xerox PARC, and just 1 year later the interactive whiteboard company SMART Technologies launched the first SmartBoard.

This product is constantly subject to qualitative changes both from a functional point of view and from the external appearance. Nowadays it is being widely used in schools making teaching very attractive and at the same time productive.

In his article "How Interactive Whiteboards took over", Kai Yoshida, in February 2021 (Yoshida, 2021), stressed that *Interactive whiteboards help many students grasp concepts much faster through increased engagement and visual learning.*

According to the International Institute for Science, Technology, and Education (IISTE), increased engagement stimulates thinking and leads to an increased personal understanding of concepts. Furthermore, according to the Social Science Research Network (SSRN), 65% of the population are visual learners, and SmartBoards can display high-resolution photos and graphics.

Numerous scholars have shed light on the use of SmartBoards and the benefits it brings to mathematical education. Back in 2013 Marice Minor, Nonofo Losike-Sedimo, Gary Reglin, and Otelia Royster in their article brought facts that: *With SMART Board, learners do not only acquire critical reasoning skills that are vital to solving pre-algebra problems, they also actively engage themselves in the pre-algebra learning process wherein this learning is viewed as processes embedded in cognitive and social contexts.*

Through socialization, the pre-algebra learners use the SmartBoard as a cognitive tool to perform and assist each other to be successful in the classroom. (Minor, Losike-Sedimo, Reglin, & Royster, 2013)

Forming and consolidating students' critical and logical reasoning in mathematics is a challenge for every mathematics teacher.

Finding different ways, beyond what books preach, and supporting students' mathematical formation with technological tools, which motivate and at the same time personalize their learning, are parts of teachers' daily routine in the teaching process.

In learning environments centred only on the course books, the students cannot fully acquire the skills of making predictions and judgments, intuitive thinking, getting motivated, doing experiments, discerning the experiment results, and extracting formulas from these results. However, material-based learning environments enable the acquisition of these skills. (Gunduz, Emlek, & Bozkurt, 2008)

The wise use of technological tools not only enhances mathematical skills related to the development of thinking in general but also develops key mathematical competencies.

One of these competencies is problem-solving. Today's learning through active methods in mathematics goes beyond learning how to solve problems, it aims to establish mathematics learning as a whole as a process based on problem-solving. *Problem-solving is not only a goal of learning math but also a key tool for doing so.* (Klerlein & Hervey, 2000)

On the other hand, moderating the role of the teacher not to offer ready-made mathematics, but to teach students to build mathematical knowledge plays an essential role on the road to success in this field. In this context, the teacher should select mathematical problems which stimulate critical thinking and mathematical reasoning in such a way that the knowledge acquired in the school benches resists time.

SmartBoards are highly efficient technological tools that provide maximum attention to the learning process arousing interest, not only in students who have a visual learning style but also in those who have other learning styles. Its role is indisputable, especially in memorizing mathematical facts.

Memorizing them is achieved through pictures, illustrations, and animations.

With the help of dynamic geometry multimedia software in SmartBoard (SB), a kind of digital whiteboard (DWB), the effectiveness of teaching and learning with the effect of animation on demand in the classroom is improved.

This is a dual delivery of geometric concepts by texts, narrations, and words accompanied by pictures, illustrations, and animations (Leung, 2008)

Various researchers have studied many variables, ranging from cognitive-emotional to high results in the learning process. Thus in the article "The Effect of Smart Boards on the Cognition and Motivation of Students", Nitza Davidovitch & Roman Yavich emphasized that:

The research findings illuminate the contribution of technology to teaching, through a case study of smart boards, in the dimension of clarity, found by the study to be a significant criterion of good teaching (Davidovitch & Yavich, 2017)

Three characteristics that transform the SmartBoard into an efficient pedagogical tool are:

1. Divergent learning — the ability to skip from pages on the screen to the internet in a structured and fluid manner. This ability simulates the associative organization of the student's brain and contributes to the organization and clarity of the lesson as perceived by the student.
2. SmartBoards serve as a cognitive tool that expands students ' minds and facilitates supported joint thinking. Since some of the mental load is transferred from the students to the board, they are free to engage in higher thinking processes.
3. Interactive learning — smart boards enable interactions between study contents and the students themselves, both face-to-face and online (Blau, 2011).

Due to the great role it plays in the teaching of mathematics in many countries of the world, studies have been conducted that have to do with the preparation of mathematics teachers, capable of implementing technology in teaching.

Researchers see the preparation of math teachers as a very complex process. *Preparing teachers to teach mathematics is highlighted by its complexities. What technologies are adequate tools for learning mathematics? What about teacher attitudes and beliefs about teaching mathematics with technology? What are the barriers?*

These questions and more frame the challenge for the development of a research agenda for mathematics education that is directed towards assuring that all teachers and teacher candidates have opportunities to acquire the knowledge and experiences needed to incorporate technology in the context of teaching and learning mathematics. (Niess & Garofalo, 2006)

In his study "The Impact of Using SmartBoard in Teaching Mathematics on the Achievement of Eighth Grade Primary Students in Gulf of Aqaba in Jordan" author Wafa 'Abdulrahim AlQuabe'h, underlined that: *the method of education through the SmartBoard is effective and positive, because it takes into account individual differences among students, in addition to that it helps to increase the interaction of the student with the educational process and increase the participation of students in the classroom, which helps to understand the educational material and intake it, and this helps to increase the student's achievement of concepts and terminology in mathematics.* (AlQuabe'h, 2019)

1.3 The problem-solving is a key competence for learning mathematics and an important personalisation method.

The culture and skills a student has in mathematics are best defined by the way he or she reasons to solve problems. The way we educators organize all our learning activities to solve mathematical problems, starting with the simplest ones and culminating in mathematical problems modelled on a real-life situation, determines the success of our students in mathematics.

These activities, combined with the care to personalize the teaching of mathematics as well as finding the appropriate technology to support and strengthen the whole process of mathematical reasoning until the final solution of problems constitute the most important tools that influence the formation of a life-sustainable mathematical culture.

Problem-solving lies at the heart of the conceptual framework of the mathematics program in pre-graduate education.

This position of it can be explained by the following two arguments:

- First, mathematical thinking plays a central role in solving real-life problems and developing modern science and technology, as well as providing basic research techniques.
- Second, the power of abstraction in problem-solving, logical reasoning, the beauty of proof, and basic techniques make it a source of pleasure, aesthetics, and motivation.

Everything in modern mathematics begins, elaborates, and culminates with problem-solving.

The learning situation that precedes math classes is in itself problem-solving. It is broken down by discussions and debates in the classroom to understand the mathematical concept and then followed again by various mathematical problems that reinforce the concept by organizing the class into groups according to their level.

These levels range from the basic level which justifies and solves simple mathematical problems that apply the concept taken in the classroom, to the models of independent work that require a high degree of reasoning and creativity that belong to the level.

So, teachers can use mathematical problems to introduce a new concept, research, discover and illustrate mathematical facts, delve into theoretical materials, build mathematical skills and competencies, encourage students to learn more, to develop their creative, analytical, and rational critical thinking.

Regarding the importance of problem-solving in mathematics Paul Zeitz in his book "The art and the craft of problem-solving" (Zeitz, 2009)states: *Someone who learns how to solve mathematical problems enters the mainstream culture of mathematics; he or she develops great confidence and can inspire others. Best of all, problem solvers have fun; the adept problem solver knows how to play with mathematics, and understands and appreciates beautiful mathematics.*

The main challenge for mathematics teachers of all levels of education is the development of critical and logical-mathematical thinking, creating a solid basis for thinking that goes beyond routine, basic practice, to reinforce basic concepts in what will call the major power of solving the mathematical problem.

Problem-solving in mathematics requires the sophistication of mathematical thinking as a whole and this sophistication is gradually built day by day, built in a personalized way, advancing according to one's needs and abilities.

The whole process should be oriented in the right direction by creating favourable conditions for advancement by delving into reasoningand creating a "climate" that promotes and develops mathematical thinking during the problem-solving process. this whole process means:

1. The necessary scientific information service for research if required by students.
2. Creating appropriate conditions for the investigation of the problem, including the necessary instructions, use of ICT, theoretical assistance, etc...
3. Promoting an inter-student and teacher-student mathematical dialogue, where the centre of the process is the student.
4. Encourage students to think of different ways of solving the problem, why not even by giving him an initial indication?
5. Encouragement to return to the solution again in case of "failure" and in case of success.
6. Asking "intelligent" questions at the right time and encouraging them to think at other times.
7. Receiving without evaluation the right or wrong answers.
8. Understanding the appropriate moment to intervene or withdraw to let students think, "rise", "fall", and "rise" again.
9. Evaluate their original ideas that come from a continuous effort of investigation and reasoning.
10. Encourage generalizations, if possible, about the problem that has been solved.

On the other hand, one of the key points of the mathematics teaching process is finding methodological educational tools and creating optimal environments that awaken, and then increase and strengthen students' interest in mathematics.

In the variety of technological tools that exist, the SmartBoard is a technological tool that, by its effectiveness, motivates students to learn mathematics by supporting problem-solving, as one of the key points of the competency curriculum.

Since problem-solving tops the list of skills and competencies to be achieved through math learning, developing math learning through problem-solving doubles the importance of learning math. Problem-solving is a very complex mathematical process and, like all mathematical processes, it is essentially the way to apply mathematical competencies creatively in previously unknown situations as well as the degree of innovation and ease of adoption in problem-solving.

As a mathematical process, the place of problem-solving is found in one of the regular tetrahedron peaks with 3 other peaks being mathematical logic, reasoning, and communication.(Figure 41).

As personalized learning in mathematics is the challenge of the 21st century, as part of the

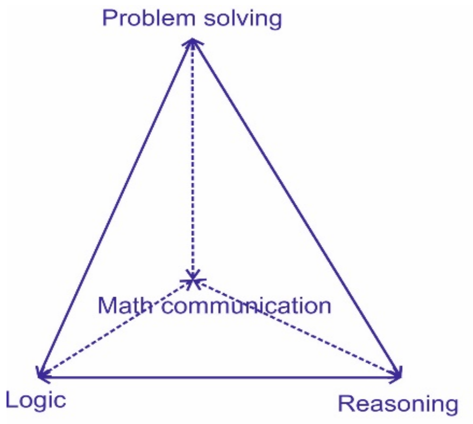


Figure 41Tetrahedron of problem-solving

personalization of all walks of life, brought about through problem-solving which is also a key competence for learning mathematics, it takes on special importance.

And if this "mixture" with fully defined ratios: (personalized learning: problem-solving), adds another element that should characterize the learners of the 21st century, which is the use of technology, can prepare generations who are not only capable of learning mathematics, but also produce mathematics being the forerunner of technological change to make their implementation in every field of life.

The world is changing rapidly in all areas, but especially in technology. In this way, education policies around the world are undergoing constant change to educate generations to meet the challenges of life and to advance by anticipating the great changes that are taking place in all areas, but especially in technology...

Orientation policies related to the teaching and learning of mathematics around the world have clearly and far-sighted defined future teaching and learning practices, based on clear ideas and principles to make them more resistant to rapid technological change. , and not only, occurring in the world. But while mathematics is the pillar of education in general, problem-solving is the pillar of mathematical learning. On the other hand, technology plays the role of empowering learning practices by enabling forms of active learning, as well as promoting and activating these transformative approaches to the future-oriented learning process.

1.4 Some perspectives on personalized math learning.

The evolution of each student in mathematical terms should be seen as closely linked to the personalization of learning mathematics, to the way we offer the mathematical concept, to the opportunities we give them to grow from the point of view of mathematical formation while increasing commitment and their desire to deal with mathematics.

From this point of view, it is very important to be careful to work based on the strengths of personalized math learning as well as to improve as much as possible its weaknesses.

Regarding personalized learning, different researchers give the following opinions:

At its most basic level, personalization consists of teachers' efforts to respond to discrepancies between students in the classroom. *Every time a teacher addresses an individual or small group to change his or her teaching to create the best possible learning experience, that teacher is differentiating the lesson.* (Tomlinson, 2000)

Nowadays the range of activities that a teacher carries out to personalize learning can be strongly supported by technological tools. It is enough for him to have access to his school, so there is a good material base, and his skills in the digital field are at the right level. But both of these conditions require it to be always updated with the latest in the field of ed-tech.

Personalization is a learning strategy that *offers different paths to understanding content, process, and products, taking into account what is appropriate given a child's profile of strengths, interests, and styles.* (Dixon, Yssel, McConnell, & Hardin, 2014)

If the personalization process begins with recognizing his / her learning profile (PPL), designing personalized plans, and searching for opportunities to adapt to their pace, the method that enables them to understand and learn the concept, success is guaranteed. Personalized teaching means creating *responsible guidelines to meet the unique needs of students.* (Clarke & Watts-Taffe, 2013)

1.5 Why was problem-solving chosen to see the benefits of teaching math through SmartBoard?

The solution to the problem is not simply a matter detached from the mathematical context but is an important part of it, not to mention the most important.

For NCTM, *Problem-solving means engaging in a task for which the solution is not known in advance. A task does not have to be a word problem to qualify as a problem - it can be an equation or calculation students, have not previously been taught to solve* (Joyner & Reys, 2000).

According to this definition, even in the chapters where there are not many mathematical problems with words, such as the chapter on function or statistics in the seventh grade, the teaching of mathematical concepts based on a problem was used, while in other chapters problem-based learning.is combined with problem-solving learning according to George Polya by observing how students acquire problem-solving skills by personalizing teaching through SmartBoard.

The stages of solving a mathematical problem have been defined by Georg Polya since 1945, but they have been used much earlier than 1945. They have certainly been used with the same efficiency since ancient times.

According to Georg Polya, the four stages of problem-solving are (Polya, 1945)(Figure 42):

1. Understand and research the problem carefully.
2. Finding a suitable strategy or plan for solving it.
3. Use the strategy or plan of the solution you found to solve the problem.

4. Go back to the beginning and reflect on the solution found.

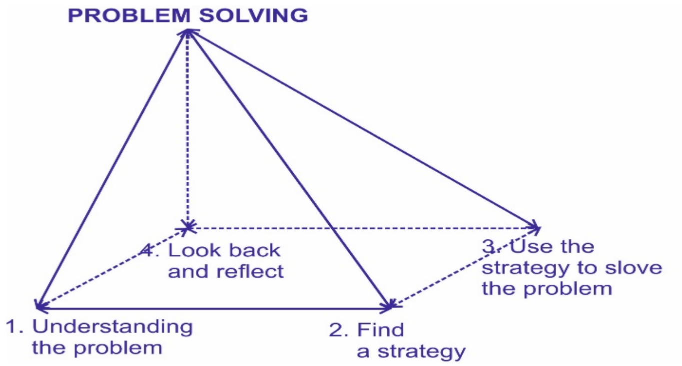


Figure 42. Four stages of problem-solving

Teaching students how to solve problems is a very rewarding investment to orient them toward problem-based learning. *Problem-based learning uses problem-solving, but it is much more than just solving problems. Problem-based learning is a broad curricular approach, it is the first problematic approach, it is integrative across disciplines and it is metacognitive in its form of assessment.* (Jefferson, 2001)

The aim is to give students the ability to solve mathematical problems that go beyond the level of routine text problems. In the initial stage of learning the concept, these problems can be considered complex problems, but with the acquisition of the skills to solve them, they turn into very simple routine exercises.

The variety of mathematical problems is very large. It varies from very simple problems, the solution of which is done with only one action, and then to problems with two or more actions, which are integrated problems. The latter requires the solution of simpler problems in advance.

On the other hand, based on the initial conditions there are problems with a given condition or problems without a given condition that are a little more difficult than the first ones.

Although there can be no fixed division between "simple" and "complex", there are problems that seem very simple, often putting students in difficulty. Judging by student levels, that problem that seems difficult for one student, for another student, that can be gifted and talented, may seem like a very simple exercise.

Math teachers should not allow themselves to get to the point where students face seemingly simple problems, which they can not solve, seeing their failure in math.

Teachers need to model and manipulate problem-solving behaviours so that when they encounter real-life problems that they have not consistently practised in school, they will be able to find a solution.

The math teacher can not provide solution models for every problem in real life, but he has the responsibility to teach students the methods of dividing a complex problem into simpler problems to teach them to think critically,and logically to find the optimal path that leads to problem-solving.Use of SmartBoard to support problem-solving, for students with different learning styles, for seventh graders.

1.6 Use of SmartBoards to support problem-solving, for students with different learning styles, for seventh graders

In 2020, at the EDULEARN conference, I presented a scientific paper entitled "The role of ICT in improving problem-solving in teaching mathematics" concerning an experimental study that I had conducted with two classes in two different schools in the city of Elbasan. From this study, it was concluded that:

The SmartBoard played an important role which was also shown in the final test results of both groups of students taken into study. It served as an innovative tool with a high potential to model problems and solution strategies, demonstrate the solution, and make necessary sketches that from a visual point of view stimulate analytical and critical thinking. As a result of this collaboration, the mathematical conceptual understanding was significantly improved and students were motivated to be an active part of problem-solving discussions.What was observed in the experimental group was a good climate of cooperation between the teacher and the students.

1.6.1 Characteristics of the study and the results obtained from it

The experiment was conducted for 8 weeks, with one seventh-grade class(experimental group) in a private secondary school in Elbasan in Albania and the results were compared with those of another seventh-grade class(control group) in a secondary public school in Elbasan.

The math teachers in both schools followed a common plan during these 8 weeks (32 teaching hours) and used the same resources and didactic materials.

The aim was to see the impact that the use of SmartBoard brings in solving problems for the development of mathematical competencies, compared to traditional teaching and learning.

In both groups, the methods and techniques used were the same, except that in the experimental group the teaching was supported by the use of SmartBoard Notebook software, while in the control group it was based on pencil and paper as well as the blackboard.

The results of the experiment showed that the students of the experimental group were more interested in discussing, listening, observing, reflecting, investigating, and reasoning by becoming an active part of the lessons. They appeared motivated and interested in learning.

The SmartBoard served as an innovative tool with a high potential to model problems and solution strategies, demonstrate solutions, and make necessary sketches that visually stimulate analytical and critical thinking.

SmartBoard strongly supported teaching and this was reflected in the final test scores, which tested the mathematical skills and competencies developed during these 8 weeks translated into learning outcomes.

In the experimental group, there was a favourable climate of cooperation between teachers and students.

Since the first experiment was conducted in a small private school, the repetition in a school with more classes would shed light on the veracity of the results obtained in the case of the experiment presented in Edulearn.

The experiment was performed again, in the third quarter of the school year 2020-2021. This term lasts 40 lessons (45 minutes each), ie 4 hours per week, for 10 weeks.

The experiment was developed with a sample 3 times larger. The experimental group (79 students in a secondary non-public school, "Vinçens Prendushi" in the city of Durrës) and the control group (60 students, in a secondary public school, "14 Novembre" in the city of Durrës. The number of students in the three seventh grades (Experimental group) is 30% higher than the number of students inthe three seventh grades (Control group).

The difference between the way the teaching took place in the two groups was that in the non-public school, the personalization of problem-based mathematics teaching was supported by educational technology, mainly by the use of SmartBoard Smartnootebook Software. The non-public school, "Vinçens Prendushi", which is part of the network of Catholic schools in Albania, has a rich educational technological base and SmartBoards in every class.

In the school of the control group, the teaching took place according to the traditional way with a blackboard and chalk. The experiment took place during the 34 mathematical hours (8 weeks and 60 minutes for tests) of the third quarter of the 2021-2022 school year.

The teacher who experimented was pre-trained by me, on how to use SmartBoard in teaching mathematics.

Both teachers (of the experimental group and the control group), followed **the same thematic plan** and **the same activity plan,** which was planned.

It was agreed that both groups should rely on the development of learning **from an auxiliary material**, in which I had created and designed a package of math exercises and problems according to student levels, emphasizing the mathematical competencies that each group of problems intended to develop.

Differentiation at the level of exercises and problem aimed to help personalized learning in both groups. The teachers were also provided with a plan of activities that they would develop each week, by the topics that they would develop.

1.6.2 Teaching/learning design

1.6.2.1 The plan of the activities in the experimental group

The teacher took notes and observed the progress of the activities defined for each week according to the following plan.

№ of the week	Targeted activity to be observed	Topic elaborated	Description of the activity developed in the experimental group
1	Analysis of a mathematical problem	Probability	The keywords of the problem text, pre-written on the computer in red, are displayed on the SmartBoard. Problems to be solved belonged to all three levels. In various problems, there was redundant and missing data. A detailed analysis of them was made. Students explained the

			mathematical meaning of the words and showed the connection they had with each other in the context of the problem. The variety of actions performed by the SmartBoard, especially with the different coloured dashes sketches as well as the storage of the material that was used in a previous document to be revisited in another step of solving the problem was the obvious advantage that stood out this week.
2	Construction of the rationale scheme.	Function	The students first found the keywords of the problem, showed the connection they had with each other within the problem, and began to build the scheme of reasoning. Math shapes on the SmartBoard were used to construct reasoning schemes and this enabled students to react more quickly to create a detailed solution plan. What was noticed for this week was their enthusiasm to build math-shaped schemes as well as the desire to build schemes on the SmartBoard. This week's feature was the activation of the GeoGebra software on the SmartBoard to illustrate the topics about the function chapter.
3	Solve the problem by going through the steps of the first two weeks in advance.	The fractions and the percentages	This week the problem-solving procedure was observed by analyzing the relationship between data and requirements and constructing appropriate reasoning schemes. This week it was noticed that the students combined all their skills as well as digital competencies to create a perfect interaction with the SmartBoard. The illustration of colour schemes and stripes significantly increased their activation in the lessons. The connection of the concepts "part" and "whole" as well as their relationships in problem situations were illustrated through schemes from math shapes and conveyed interest to students of all levels. On the other hand in terms of the concept of percentage, it was treated analogously to fractions given fractions with denominators 100.
4	Theway students review the problem-solving	The fractions and the percentages	This week all the problem-solving steps were reviewed from the beginning, to reflect, on why not to think differently. Favoured by the Smart Board's ability to store previously used documents they had the opportunity to return and review the

	procedure was studied.		procedure, scheme, illustrations, and solutions made. The database created in the first weeks served to speed up the process as well as to save time that could be wasted if we brought them back to the blackboard from scratch.
5	"Puzzle week" was dedicated to the study of how students have fixed steps to solve the problem.	Statistics	In the fifth week, the students were given solved problems in which parts of the solution or the order of the solution stages were intentionally confused and the students were asked to put them in the right order. The lesson took place in the computer lab where each student had a personal computer. Pre-prepared texts were displayed on each student's SmartBoard and personal computer. Students desire to complete the sequence of solution steps in record time was noted, as well as a desire of students to present the solution demonstration on the SmartBoard.
6	The sixth week was devoted to thinking week differently.	Ratio	This week focused on problems that had at least two possible solutions. Students were given problems through which the aim was to investigate the diversity of solutions and to present and justify them. After solving the first problem (the solution most of them thought of) the students were asked to reason for a different solution, because as Polya said - "It's Better to solve a problem in 5 different ways than 5 problems in the same way". in some cases, when students were late in giving a different solution through an illustrative scheme, they were asked "questions" as indications to lead them to the solution. For the lower-level students, if it was seen that this indication did not allow them to start the solution, they were given another indication, constantly asking them to judge and start reasoning logically and critically.
7	The seventh week was devoted to creativity.	Proportion	The seventh week was dedicated to creativity. In the seventh week, the students were given the keywords or the solution scheme for different problems, and those in mixed groups according to the levels created and solved problems as desired. The SmartBoard served to present the ways of reasoning, the solution scheme, and the final solution to the problems to each group. The works of each group were preserved and were

			the subject of discussion when the groups presented the work.
8	The 8th week was dedicated to the "Competition with the most beautiful problems"!	Measuring time	This week's theme was actions with time units. The students created problems where the knowledge gained on the units of time measurement with context from real life was applied and a team selected by them as a jury, chose the most beautiful problem. (The criteria for the most beautiful problem were first set by them. This week was a higher rate in Bloom's taxonomy than the seventh week, as students during this week created and solved problems with context from real life.
9 (only 60 minutes)	Test	The most thematic problems from probability, fraction and percentage, ratio and proportion, and units of time measurement.	The way of solving the problems around these topics was observed and it was compared with the progress of problem-solving in the second term. On the other hand, a comparison was made of the learning outcomes in the third term quarter compared to the second term.

The above plan was developed and implemented based on the following (described in the next section) exercises and problems where the topics they belong to are defined as well as the mathematical competencies that they develop according to the level of the students. The experimental group used the SmartBoard with all the benefits that come from its use. The teacher and the students of the experimental group often shared files, used resources from the Internet, and used software to build different graphics.

The variety of SmartNootebook tools, including ready-made geometric shapes, and geometric tools to build figures and graphics enabled students to visualize, sketch, and colour schemes of solutions to different mathematical problems.

The lesson with SmartBoard was more dynamic and drew the students' attention with illustrations through the tools it possesses. Finding preliminary information on the web and sharing it in class made the lesson interesting and increased the students' interest.

1.6.2.2 The additional package of problems and exercises, that supported the whole process in the experiment

The whole process was supported by SmartNootebook of SmartBoard. The teacher who experimented was supported by auxiliary material, mainly mathematical problems whose solution, was aimed at developing mathematical competencies.

The package of exercises and problems was the same for both groups. In the experimental group, students' solutions and graphic illustrations were also sent home on the Googleclassroom platform, so students could review the material.

Equipped with diverse content options, teaching materials are stored in designated files for each lesson. Sending these files to each student's account on the Google Classroom platform also helps students with difficulty learning the "weight" of anxiety related to the time limit. The experiment took place during the time of the restrictions of the pandemic and often the teacher

included students with health problems in video conferences, without the need for specific cameras. This made it possible to sometimes develop hybrid learning.

A package of exercises and problems, with instructions regarding the competencies that each of them will develop, as well as explanations about the pedagogical apparatus of questions, was an auxiliary material, which supported and guided the work of both teachers, in both groups.

The package was drafted with the thematic plan and supported the plan of activities. In the set of questions, there are exercises and problems for all levels of students and their level of thinking.

This material is an auxiliary material, which served as a supplementary source to the textbook "Mathematics 7, for Cambridge Secondary 1" by Patrick Kivlin, Sue Pemberton, and Paul Winters (Pemberton, Kivlin, & Winters, Mathematics for Cambridge Secondary 1, Stage 7, 2014).

1.6.2.3 Analysis of the mechanism of questions developed for problem-solving.

Asking questions is a key part of teaching math. The way questions are formulated, how they are differentiated, what is intended through them, and how they affect the development of skills, values, and attitudes, makes the learning process unique. They vary from one class to another, from one teacher to another, and depend on both the students 'academic level and teachers' ability to ask questions that stimulate critical thinking and aim at mathematical reasoning and argumentation as key elements in teaching mathematics. But, what remains constant, is that to personalize the teaching of mathematics the questions must meet these conditions:

1. To be as diverse as possible.
2. To have different levels.
3. To be asked at the right time and in the right place.
4. Alwaysaim at developing key competencies in the subject of mathematics.

For rational reasoning, through which a final solution to the problem is aimed, it is very important that in the **first step**, the basis of the questions that will be asked to fully reveal the connection between the data and the requirements of the problem and to shed light on the concepts. mathematical to be used in its solution.

Asking focused questions, and finding connections between mathematical concepts that support problem-solving is the **second step**.

Researching the level of their reasoning through the questions: ***How?***, ***Why?***, ***What?*** etc. is the **third step.**

And the **fourth step** is the ***Thought Fair***, where students are stimulated through questions that promote the comparison of solutions, seek new solutions, seek to investigate if there is any other possible solution, etc.

Based on this scheme, students of both groups develop reasoning by solving exercises and mathematical problems by the mathematical competencies that they would develop according to the thematic plan of this semester.

Below are the exercises and problems of this help package, as well as the mathematical competencies that aim to develop their solutions.

1.6.3 Experimental topics

1.6.3.1 Topic: Probability (first week)

Thematic plan:

- Probability and its meaning
- The values of probability. How to find the probability?
- Probability estimation and experimental probability.
- Reinforcement of knowledge on probability.

The mechanism built to ask students and build new knowledge from them was based on a combination of techniques and methods where the steps of solving the problem, followed the Polya steps, while the question mechanism was designed according to the level of thinking based on the taxonomy of Bloom (Bloom B. , 1956), but redefined by (Sanders & Sandler, 1966), which defines these 7 levels of questions as follows:

1. **Memory**: The student recalls or memorizes information
2. **Translation**: The student changes information into a different symbolic form or language
3. **Interpretation**: The student discovers relationships among facts, generalizations, definitions, values, and skills
4. **Application**: The student solves a life-like problem that requires identification of the issue and selection and use of appropriate generalizations and skills
5. **Analysis**: The student solves a problem in the light of conscious knowledge of the parts of the form of thinking.
6. **Synthesis**: The student solves a problem that requires original, creative thinking
7. **Evaluation**: The student makes a judgment of good or bad, right or wrong, according to the standards he values.

Problems and exercises, and corresponding didactic comments:

1. In a bag are 4 red spheres, 3 blue, and 2 white.
   a) Find the number of elements of the resulting space in the random sphere selection experiment.
   b) Find the probability of the event:
   – "The selected sphere is red".
   – "The selected sphere is blue".

– "The selected sphere is not white".

Mathematical competence that developed: **Conceptual connection and mathematical thinking.**

The situations from ***a)*** and***b)*** are built based on the application of mathematical processes recalling knowledge of probability.

2. In one box are 5 yellow and 11 red pencils. A pencil is randomly selected from the box.
   a) What is the probability that the chosen pencil to be red?
   b) The pencil does not return to the box.What is the probability that in the second choice we will have a red pencil again?

Mathematical competence: **Mathematical reasoning and validation.**

The student builds a chain of reasoning, which starts from finding the space of results in the request a), to finding the probability in b), where the logical organization of the given facts and conditions is done beforehand and then the final solution.

3.In a box are 50 floppy disks with numbers from 1 to 50. A floppy disk is selected randomly. Calculate the probability (expressed in percentage) that the ejected floppy disk has a number that is:
   a) A multiple of the number 6.
   b) Divisor of number 28.
   c) A number, ending in 4 and located between the numbers 12 and 42.
   d) A number, less than or equal to 30.

4.A neighbourhood has 52 houses, numbered from 1—52. One of these houses is for sale. Find the probabilities that:
   a) The house for sale has 5, as a digit in its number.
   b) The house for sale is numbered 30.
   c) The number of houses for sale is even
   d) The house for sale has the number 64

5. In the set of two-digit numbers, multiples of the number 9, one of them is selected randomly. Calculate the probability that the number obtained is a multiple of the number 27.

6. A box contains 50 small balls with the numbers 1 - 50. A ball is randomly taken out of the box. Find the probability that the received ball has:
   a) A number ending in 6, included between the numbers 15 and 45.
   b) An even number is included between the numbers 15 and 45.
   c) A number that is divisible by 7, is included between the numbers 15 and 45.

Mathematical competencies that aim to develop problems 3-6 are:**Mathematical reasoning and validation, conceptual connection, and mathematical thinking.**

Problem-solving, answering its questions, designed with requests which come in handy, aims to bring to attention the knowledge of natural numbers and knowledge of the multiplier of the divisor, in parallel with the development of the concept of probability. A close conceptual connection between this knowledge of numbers as well as knowledge of probability leads to the final solution to the problem.

7. (Gifted and talentedstudents)

In a class of 38 students, 20 are studying English, 12 are studying French, and 6 are studying English and French. Calculate the probability that one student selected randomly will be:

a) A student that studying **only** English.
b) A student who studiesEnglish and French at the **same time**.
c) A student that studying **only** French.

Mathematical competencies that develop this problem are**Problem-solving, Reasoning, and mathematical verification.**

This problem aimed, among other things, to emphasize the mathematical meaning of the boldest words in the text of the problem and by initiating the construction of a scheme that belongs to the intersections of sets, which as a concept advances to higher classes.

8.(Gifted and talented)

Three athletes A, B, and C are in a race. Athlete B has 3 times more chances than A to win the race and athlete C has 3 times more chances than athlete B to win the race. What is the probability that each athlete wins the race?

9. (Gifted and talented)

A bag has white, red, and black spheres. The ratio of the probability of choosing randomly a red sphere to that of choosing a white sphere is 3: 4. The ratio of the probability of choosing randomly a black sphere to that of choosing a white sphere is 3: 5. What is the probability of choosing a coloured sphere:

a) White?
b) Red?
c) Black?
d) How many spheres at least can be in this bag?

The mathematical competencies that develop problems 7-9 are**Problem-solving, conceptual connection and mathematical thinking, mathematical reasoning and validation.**

These problems aim at the intellectual development and creative intuition of the students, as well as the differentiation of the content, to personalize the teaching.

10. Alban prepared tea or coffee for his friends. He drew a chart as follows where he threw the number of drinks prepared by type.

The type of drink	The number of drinks prepared
Coffee with milk	3
Coffee with milk and sugar	2
Milk tea	4
Tea without milk	1
Tea with milk and sugar	3

a) If one of the friends takes a cup by chance, how likely is it, that he gets a cup of tea with milk?

b) If his other friend also wants to get a cup of milk tea, what is the probability that he gets what he wants when it is known that the first friend got one?

This problem aims for the student to build a chain of reasoning, interpreting a practical mathematical situation, thus developing the **competence of mathematical reasoning.**

11. Numbers are marked on the sides of a straight homogeneous dice: 1, 1, 5, 5, 5, 6.

Blendi rolled the dice 80 times and scored the results on the board.

a) What is the theoretical probability that number 1 will fall, when the dice are rolled one time?

b) What is the experimental probability that number 1 will fall?

c) What is the theoretical probability that the number 5 will fall, when the dice are rolled one time?

d) What is the experimental probability that number 5 will fall?

The results	Frequency
1	27
5	40
6	13

12. A straight homogeneous dice, with 6 sides, where the numbers from 1 to 6 are marked, is rolled 60 times. The results are tabulated.

a) What is the experimental probability of each of the numbers falling?

b) Compare each of the experimental probabilities with the theoretical ones, when the dice rolled one time.

The results	Frequency
1	12
2	13
3	8
4	10
5	10
6	7

13.A bag contains red and blue beads. One is randomly selected, and the colour is marked and returned to the bag. The results of 40 selections are listed in the table.

The colours	Frequency
Red	13
Blue	27

a) Estimate the probability of choosing each colour.

b) If there are 6 beads in the bag, how many of them do you think might be red?

14. After that, if we continue with 20 more elections, the results are as follows:

a) Estimate the probability of choosing each colour now.

b) Which prediction is more accurate? Why? Discuss.

The colours	Frequency
Red	8
Blue	12

Problems 11, 12, 13, and 14, aim through a comparative analysis of theoretical and experimental probability to logically organize the facts achieving the appropriate result. All this activity aims to develop mathematical reasoning and validation.

On the other hand the problems "translate" the mathematical solution into the solution of the situation taken in the real context, developing the competence of mathematical modelling.

1.6.3.2 Topic: Function (Second week)

Thematic plan:

- Function
- Horizontal and vertical lines
- Equation of the line
- Exercises and problems with the equation of the line

The function is an important concept in school mathematics and its treatment is very delicate.

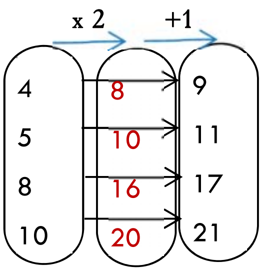
– The seventh-grade mathematics curriculum expands and takes its form of treatment with the right scientific rigour, as well as the right terminology, compared to the concrete examples that prevail in the lower grades, treated through "machines of numbers", which played the same role.
– Confronting students for the first time with a series of new concepts in such a short time complicates the learning process.

The following exercises and problems aim to make this transition as smooth as possible and to absorb the concept with due scientific accuracy.

Problems and exercises, and corresponding didactic comments:

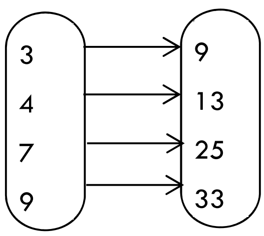
1. Based on the values given in the domain and the range, find the rule according to which the elements are related.

a.Domain Range



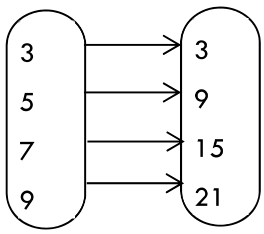
The rule:x → ....

b.Domain Range



The rule:x → ....

c. Domain Range



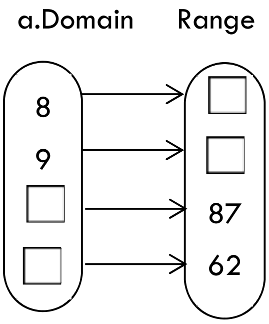
The rule: x →........

The exercise aims that students after analyzing the relationship between the responsible elements of the set of values for each of the two consecutive elements of the starting set define a ***transit*** set, from which they would go to the set of values as in the example 1.

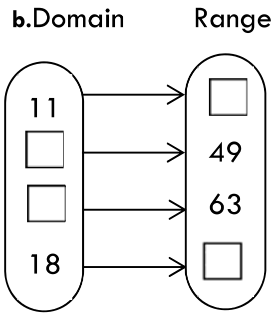
The illustrations with colour sketches on the SmartBoard, as well as the explanation of the procedure, make the difficulty of the exercise lessen in case b).

Case c) belongs to a slightly higher level of reasoning, so it is recommended for gifted students. Develops mathematical reasoning and argumentation and promotes mathematical thinking.

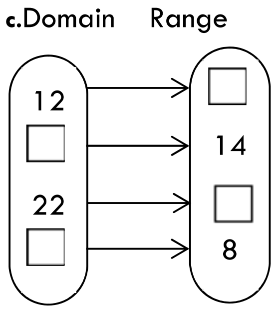
2.Complete the diagrams below:



$x \to 5x + 2$



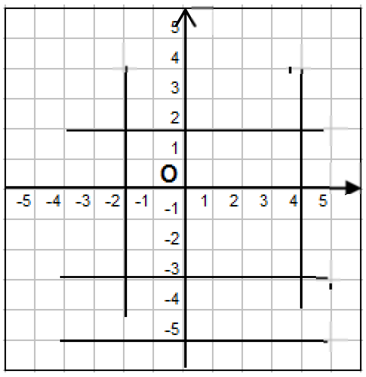
$x \to 7(x - 5)$



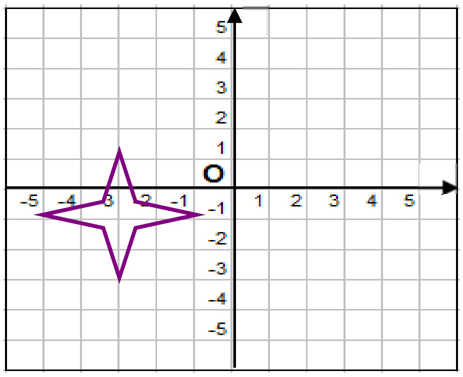
$x \to (x-7):5$

The solution of exercise 2, by completing the face-simulation pairs, simultaneously connects the concepts of finding the numerical value of an expression when the value of its variable is given and solving the equations of the first power with a variable. Develops the concept of conceptual connection and mathematical thinking.

3. Write the equation for each line in the graph



4. Write the equation of the mirror lines for this figure.



Exercises 3 and 4, which seek to find common features that have points found in the given lines and by logically organizing the facts, develop the competence of mathematical reasoning, even in exercise 4, students recall the mirror line.

5. Given the coordinates of the points:A*(- 4; - 3), B (- 2; - 3), C(0; - 3),D(2; - 5), E (5; - 3).* Find which of the points **is not in** the same line as the others.

6.

a) Present the points in the coordinate grid:*A (−4; −2), B (2; −5), C (4; −2),* and *D (2; 1).*
b) Name the quadrilateral *ABCD*. Show its properties.
c) Write the equation of the sides and diagonals of the quadrilateral *ABCD*.

Exercises 5 and 6, based on inquiry and reasoning, aim to develop the **conceptual connection** and **mathematical thinking** of students, as well as **mathematical reasoning** and **skills for validation**.

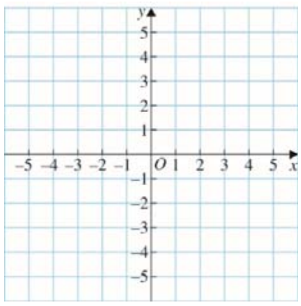
7.The following points are given: $A(1;\ 3)$, $B(2;-4)$, $C(-1;\ 11)$, $D(3;\ 7)$, $E(4;\ -13)$. Which of them lies on the line defined by equation $y = 9 - 2x$?

8.

a) What is the equation of the horizontal line passing throughpoint $F(16, -52)$?

b) What is the equation of the vertical line passing through point $H(34, 17)$?

9. On the coordinate grid, plot the points and the line that passes through them. Points: $A(3, -3)$, $B(5, -3)$, $C(1, -3)$, and $D(-4, -3)$.



a) What is the equation of the line, passing through these points?

b) What is the equation of the line passing through the points $(4; 3)$ and $(4; -1)$?

c) Plot its graph in the given coordinate grid.

d) Drawon the coordinate grid the graph of the equation $x = 1$.

e) If the points of intersection of these 3 lines with a fourth line are the vertices of a square, what is the equation of the fourth line?

In exercises 7, and 8, the pedagogical apparatus of questions as well as the escalation of thelevels of exercises reinforcethe concepts:

1. If the coordinates of a point satisfy the equation of a line, then the point lies on the line and vice versa.

2. To construct a line parallel to one of the coordinate axes, it is enough to know the coordinates of a point and the axis to which this line is parallel.

Competence in**mathematical reasoning** and **mathematical modelling** is developed.

In exercise 9, students draw with pencil and paper in a notebook, research the questions asked, and then apply the exercise to the GeoGebra application.

10. Complete the table of values for each of the functions and construct in the 2 coordinate grids, the lines with the given equations (in each grid two lines).

$a.\ y = -2x$

x	-2	-1	0	1	2
y					

$b.\ y = 5 - 2x$

x	-2	-1	0	1	2
y					

$c.\ y = 4x + 1$

x	-4	-2	0	4
y				

$d.\ y = -\frac{1}{4}x + 3$

x	-2	-1	0	1	2
y					

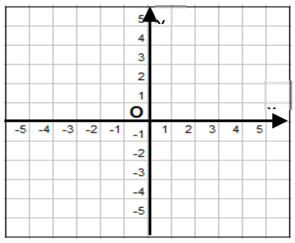
Show what you notice about the position of the two lines in each graph.

11.

a) Complete the table of valuesaboutthe line with the equation:

$y = 3x - 1$.

X	-1	0	2
Y			

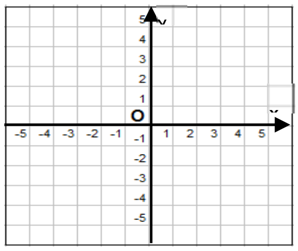


b) On the coordinate grid draw the line with the equation

$y = 3x - 1$.

c) Write the coordinates of the point of intersection of the line with the *x*-axis.

Can you find it without drawing the line? Discuss.



12.

a) Complete the table of values:

$$y = \frac{x-3}{2}$$

x	-1	1	5
y			

b) On the coordinate grid draw the line with this equation.

c) Write the coordinates of the point of intersection of the line with the axis of the ordinates. Can you find them without drawing the line?

Exercises 10, 11,12 with different degrees of difficulty, are designed to complete the framework of the construction of the line, its positioning on the coordinate grid, and its reciprocal positions with the coordinate axes. On the other hand, the concepts obtained here, are the basis for the development of knowledge on function in the eighth grade, where students have to build a straight line when they know only 2 of the points through which it passes, namely the intersection with the axis of abscissas and ordinates.

The lines are pre-built in the notebook and then a session is held in the GeoGebra application, through which students practice and build many other lines with given equations.

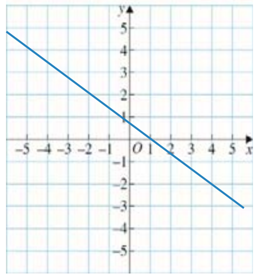
During this session, the teacher instructs students about the construction of straight lines that have different positions on the coordinate grid, including those perpendicular to the coordinate axes ($x = 3$, $y = 4$, the bisectors of squares),and pushes students into an inquiry about angular coefficients.This develops the mathematical competence of **mathematical reasoning and argumentation** as well as the **conceptual connection** and **mathematical thinking.**

13.

a) Fill in the table referring to the graph shown in the figure.

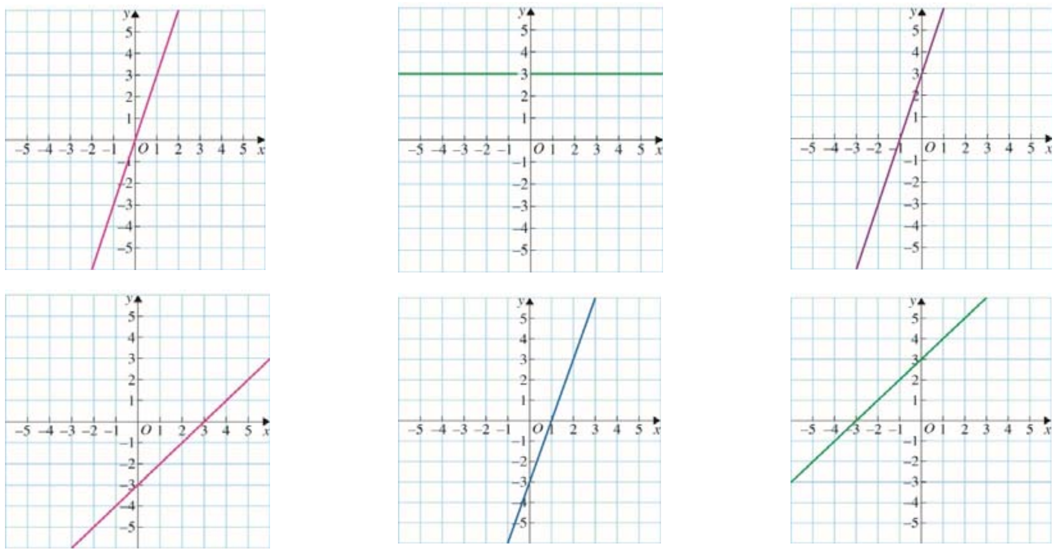
x	-5	-2	0	1
y				

b) Write the equation of the line.



14. Place each line equation on its corresponding graph.

$y = x + 3; y = 3x;$ $y = 3;$ $y = x - 3;$ $y = 3x + 3;$ $y = 3x - 3$



Exercises 13 and 14 develop students' **research skills** to **conclude.**

The expectation for these two exercises is to develop competence in**mathematical thinking, reasoning and argumentation,** and **conceptual connection.**

1.6.3.3 Topic: Fractions, decimal numbers, and percentages. 2 weeks (8 teaching hours)

Thematic plans:

- Fractions as part of the whole.
- Fractionally relating percentage and decimal numbers
- Use of percentage (2 hours)
- Finding the percentage of a quantity(2 hours)
- Exercises and problems, related tothe percentage (2 hours)

The success of students on this topic depends in particular on the level of knowledge of fractions. The gradual transition from fractions as a whole to decimal fractions, and then to decimal numbers, paves the way for a scientifically based understanding of percentages as a concept. The fact that decimal numbers and percentages are nothing more than expressing fractions differently makes this connection very strong.For this reason, students must know how to convert these three different forms to express the same number to each other.

Problems and exercises, and corresponding didactic comments:

1. Fill in the table to see the relationship that exists between fractions, decimal numbers, and percentages.

Fraction	Decimal number	Percentage
$\frac{23}{100}$		
	0,37	
$\frac{11}{20}$		55%
	0,19	
		7%
$\frac{9}{10}$		

In exercise **1.**, it is required to complete the table by converting the same number from one form to another. Such an exercise develops the competence of **conceptual connection** and **mathematical thinking**. Illustration of the percentage with grids of 100 squares and lined parts depending on the number expressed as a percentage, through the SmartBoard.

Only when students have established confidence in the equivalence between these three forms is there a guarantee that they will deepen with the concepts in this topic.

2. In the literature competition in October, participated: 15 students from seventh grade, 28 students from eighth grade, and 7 students from ninth grade.
Find what percentage of participants were from seventh grade, how many from eighth, and how many from ninth.

3. The students of a class are divided according to musical preferences into these groups:
12 students like only folk music, 15 only like modern music, and 3 students like both at the same time. Express in percentage each of the preferences.

4. In a class 9 students like math, 18 students like literature, 3 students like both, and1 student like neither.
Find what percentage of students in the class just like math, what percentage of students like only literature, what percentage of students like both, and what percentage do not like either.

Problems 2, 3, and 4, which belong to the three levels of students, from the lowest (problem 2) to the highest (problem 4), have to do with expressions in percentages of a given size while personalizing also teaching.

This final result requires a concrete analysis of the situation in each of the problems, further the construction of an appropriate scheme based on the keywords of the problem, and then the solution and its interpretation in the given context.

Reasoning schemes for each group are prepared on theSmartBoard.

So, through these problems they are developed**problem-solving competence, mathematical reasoning** and **validation, conceptual connection,** and **mathematical thinking,** as students have to interpret the steps of the problem-solving process, and must present mathematical information, using the connection that exists between mathematical concepts.

5. The table shows the nutrition of a bag of potatoes that weighs 50 grams. Fill in the table.

Ingredients	Mass		Percentage
Fats	14 g		
Carbohydrate			53,4%
Fibre	3 g		
Protein			5,5%
Salt	0,85 g		
Others			

6.I had 8,000 ALL. I spent 10% on food. After that, I spent 10% of the remaining money to pay the energy bill. And finally, 10% of the remaining money is spent on clothes. Find out and fill in the table:

	ALL	Percentage of 8000 ALL
At the beginning:	8000	100%
I spent on food		10%
The money remaining after the spent on food.		
I spent energy		
The money remaining after the spent on energy.		
I spent on clothes		
The money remaining after the spent on clothes.		

7. Find the following percentages.

a) 50% of the 80
b) 80% of the 50
c) 40% ofthe 60
d) 60% ofthe 40

What do you notice about the answers obtained in a)and b), and in c)and d)?

8.Based on exercise 7. write the following expressions differently and find their values.

a) 24% of 50.
b) 8% of 25.

Exercises 5, 6, and 7, it is intended to develop **conceptual connection** and **mathematical thinking,** as well as **mathematical reasoning and validation** (problem 7), where students have to logically organize the facts or conclusions reached and thenapply them further in solving the other exercises (exercise 8).

9.In a box of 40 pencils 3% of them are red. How many pencils are red?
How many pencils are not-red?

11. In a volleyball match, 2% of a team's points, or 18 points were scored directly by the service. How many points did this team score in total?

11. Sunflower contains 25% oil. How much oil do we extract from 5*kg*of sunflower seeds?
How many seeds do we need to get 1 *kg* of oil?

Through problems 9.,10. , and 11., which are listed from the simplest to the most complex, the aim is to find the percentage of the part of the quantity when the size is given, as well as to find the whole size when only a part of it is given. in percentage.

Here, too, the **mathematical competencies** for problem-solving, **mathematical reasoning** and **validation, conceptual connection,** and **mathematical thinking** are developed.

12. The price of a commodity decreased by 25% or by 800 ALL. What was the price of the commodityat the beginning? What is the current price? Find it in two ways.

13. A book costs 1000 ALL. At the book fair, it was sold at 20% off.
How many books can we buy, if we have enough money to buy 4 books before the discount.

14. The salary of a worker was $ 300. It increased by 20%. How many dollars did it make? What if after that she increases by another 10% on the new salary? How much is it made? What percentage is the new salary higher than the initial one?

15. Ana wants to buy a new dress.
The seller will make a 15% discount on the price of the dress. How much money will Ana pay, if she buys one of the following dresses?

a) A black dress that has a price of 5200 ALL.
b) A yellow dress that has a price of 3800 ALL.
c) Ana has 6000 ALL in her bag. She bought a black dress. What percentage of the initial sum is left?

Problems 12., 13., 14., and 15. are problems that bring about situations of percentage application in real-life contexts and that develop the student's **intellect** and **creative intuition.** They develop the **competence to solve problem situations,** but at the same time prove the importance of mathematics in everyday life. The choice of such exercises, where students touch on the role and

importance of mathematics, motivates them to learn mathematics and makes them willingly learn a subject that will serve them.

16. In a water solution with sugar there is 60% sugar. The solution weighs 5 kg.
Add another 3 kg of water. What percentage of sugar does the solution have now?

17. A 5 kg salt solution has 8% salt. How much more water do we have to add to this solution so that the salt concentration in it becomes 2%?
Instruction. (The amount of salt remains the same.)

Problems 16. and 17. connect mathematical knowledge with other sciences, such as chemistry, providing solutions to problems of this kind.

In these two problems it is very important to explain in advance the concept of the mass of the solution, as the sum of the mass of the solute with the mass of the solvent, and then move on to the mathematical interpretation of the problem solution. In all these problems with real-life contexts, the translation of the real context into mathematics is observed, followed by the interpretation of the mathematical result in the real context.

So, mathematics *takes* the problem with all its complexity, solves it, and *serves* the reality solved on a golden plate. This is where the greatness of mathematics lies.

18.(Gifted and talented) I have invested money in a company. Every year they pay me 15% of the amount I have invested. I save this money in another account. What% of the money I had saved after 8 years? Is this possible?

19.(Gifted and talented) I put in a bank 200,000 ALL. At the end of the year, the amount I had to receive had gone to 220,000 ALL.

a) Write the ratio of the amount of money that was added to me at the end of the year, with the initial amount of money. What percentage is this?
b) If I leave all the money in the bank, with the same interest rate, for another year, how much money will I get at the end of the second year? What percentage of the initial value is the profit?
c) With the 200,000 ALL, I had at the beginning I could buy 40 bags of wheat. How much did a bag cost?
d) The price of wheat rose by 5% this year. Find how many sacks of wheat you can buy now at the end of the year with the same amount of money.
e) The price of wheat increased by 15% in the second year. Find out how much a bag of wheat costs at the end of the second year. Find out how much more it costs compared to the beginning.
f) How many sacks of wheat can I buy with the money I have in my account at the end of the second year? Can I buy more wheat at the beginning or end of two years? Explain using percentages.

20.(Gifted and talented) The price of a car dropped first by 25% and then by 1/10 over the reduced value.

a) What percentage is 1/10?
b) If the car initially cost 3,000,000 ALL, find out how much the price of the car has become after two reductions that have been made.

Explain why the reduction that has been made is not 35%.

Problems 18., 19., and 20 planned for talented and gifted students, require a degree of **higher reasoning**, and the use of **synthesis analysis**. How the request is formulated, in addition to the highest degree of difficulty, requires mathematical argumentation and reasoning, linking facts, bringing to attention mathematical concepts, as well as analyzing possible solutions. From this point of view, their solution serves to develop to the highest possible degree all the competencies of the subject.

1.6.3.4 Processing of statistical data (Fifth week)

The topic of probability and statistics as one of the 5 topics studied in school mathematics, in theseventh grade, occupies about 13% of the math hours planned in the curriculum. (18 hours / 140 total hours)

Thematic plan:

- Arithmetic mean;
- Mode, Amplitude, Mediane;
- Comparison of statistical distributions;
- Exercises for reinforcement knowledge on statistical data processing.

In this class, students collect, read, understand and interpret data. The contenttreats statistics culminating in decision-making that pertains to our daily.Knowledge of data collection and presentation that have been trained in the first and second semesters of grade seventh are supplemented with knowledge ofaverage values, such as mode, arithmetic mean, median, and amplitude. In this topic, students are also introduced to the comparison of statistical distributions thus completing the full framework of knowledge on statistics, defined by the curriculum for this class.

The following exercises and problems, that supported the theoretical material or that served as learning situations at the beginning of the lesson or as exercises in the last stage of the lesson to reinforce the knowledge, together with the exercises and problems addressed in the book, aimed to develop all competencies in the field such as:

- **Problem-solving,**is by conducting observations on the range of statistical data, to calculate the average values and then interpret them, drawing conclusions and making comparisons.
- **Reasoning mathematical verification**, introducing different ways,in which a result can be reached,is required to make comparisons over average values and on how they make comparisons.
- **Mathematical thinking and communication,** follow a variety of specific question structures that promote mathematical thinking critically and logically, focusing on the role of mathematics in everyday life.
- **The conceptual connection**, seeing mathematics as unique with all its themes, and the problems designed for the experiment, clearly sought to use knowledge from all these themes by constantly recalling and bringing to attention this connection between them.
- **Mathematical modelling,**using differentschemes and graphs to solve problems that have cross-curricular links with all other subjects, as well as with important aspects of daily life.

- **Use of digital technology in mathematics,** taking advantage of the SmartNotebook application in SmartBoard students communicated, illustrated, and made reasoning schemes for interpreting mathematical information.

The most important aspect of this chapter is that it concludes the phase of studying statistical data. In this way it was intended to **raise the level of reasoning** as well as to **enrich the mathematical vocabulary** with mathematical terminology, to describe different situations from real life, focusing on the gradual transition **from concrete illustrative thinking to abstract reasoning.**

Let's look at the mathematical competencies they develop and the purpose of addressing each of the problems in this chapter.

Problems and exercises, and corresponding didactic comments:

1. Calculate the arithmetic mean of each numerical data set.
   a) 5; 7; 9; 13; 10; 11; 8.
   b) 31; 45.5; 24.2; 18; 20.5; 32.8.
   c) 420; 580; 900; 2160; 3240; 4100; 3850; 1290.

3. On a ride, a motorcyclist travels the first hour of 65 km, the second hour of 75 km, the third hour of 50 km, and the fourth hour of 70 km. Find the arithmetic mean of the motorcyclist's speeds.

3. At the end of the first semester the average grade of three seventh-grade classes were:
   - VIIA has 32 students: 8.2.
   - VIIB has 28 students: 7.8.
   - VIIC has 25 students: 7.4.

To find the average grade of the students from all three classes, Ermira did this: Add up all three grades and divides their sum by 3.

(8.2 + 7.8 + 7.4): 3 = 7.8.

Did Ermira find out right? How would you find out?

4. The following data show the number of absences (per day) made by 25 students in seventh grade.

0; 3; 3; 2; 8; 1; 7; 9; 7; 0; 2; 12; 0; 6; 10; 5; 5; 15; 5; 9; 8; 6; 25; 3; 4.

   a) Display the data in the frequency table.
   b) Calculate the arithmetic mean.

In this group of exercises (1.,2. ,3.,4.), the aim is to find the arithmetic mean in situations that differ from those of an implementing character, the formula for finding the arithmetic mean is applied to the argumentative reasoning required in exercise 3., thus developing mathematical thinking, conceptual connection as well as mathematical reasoning and argumentation.

5. The sum of 12 numbers is x, while their arithmetic mean is 21. Find x.

6. The sum of 5 numbers is 45.
   a) Find their arithmetic mean.
   b) All numbers are multiplied by 4. What is the arithmetic mean of the numbers obtained?

c) All numbers are increased, by 4. What is the arithmetic mean of the numbers obtained?

7. How does the arithmetic mean of 5 numbers change if:

a) Numbers are multiplied by 5.
b) Each number isdivided by 5.
c) The numbers are decreasedby 5.
d) What conclusions can you draw? In connection with exercises 6. and 7.

In Exercises 5., 6. and 7. students reinforce their knowledge of arithmetic mean by solving situations that correspond to slightly deeper reasoning, situations that do not have a purely implementing character but require argumentation and logical connection of the concepts taken.

8. The arithmetic mean of 4 numbers is 7. The arithmetic mean of 2 of them is 9. What is the arithmetic mean of the other 2 numbers?

9. The arithmetic mean of the grades of 5 boys is 8.2, while that of 4 girls is 8.65. What is the arithmetic mean of 9 students?

10. The sum of 12 numbers is 156. 5 of them have the arithmetic mean of 20. Find the arithmetic mean of the other 7 numbers.

In exercises 8., 9., and 10 the problematic situations that come and escalate, exercises require the students to make a diagram and reason on the data and requirements as well as the connection that exists between them. The level of reasoning is a bit higher, but not the highest level.

11. Calculate in each case the mode, median and arithmetic mean in the following statistical data.

a) 23; 32; 20; 44; 51; 36; 24.
b) 18; 10.6; 22.4; 15.3; 27.5; 30.2; 18.4; 25.2.

12. A bookseller has recorded the number of books sold during the 20 days of a month. The data are: 10; 7; 15; 8; 9; 3; 13; 6; 11; 8; 5; 12; 10; 9; 10; 5; 9; 6; 13; 10.

a) Display the data in the frequency table.
b) Find the mode, median and arithmetic mean.
c) Find the amplitude.

13. Find out:

a) 5 integers, so their arithmetic meanis 5.
b) 5 integers, so their median is 8.
c) 5 integers, so that their mode is 6.

14. Which of the following statistical data has the mode equal to the arithmetic mean?

a) 26, 36, 36, 23, 22, 25, 48, 36.
b) 18, 22, 36, 17, 36, 33, 52, 38.

15. Find the arithmetic mean, median, mode, and amplitude of this data set.

7, 7, 24, 5, 2.

We add a number to this data set. What number can it be, if:

a) Does the mode remain the same?

b) Does the average remain the same?

c) The average increases by 2.

We remove a number from this data set. What number can it be if:

d) Do amplitude and arithmetic mean to remain the same?

e) Does the arithmetic mean increase by 1?

In the group of exercises 11. – 15. the situations are of different types, they are very diverseand aimed at reinforcing the knowledge that has to do with finding the average values in each of the situations. The reasoning given in each case reinforced the knowledge of theprocessing of statistical data obtained in this chapter.

Through them, all the subject competencies are developed, especially **mathematical reasoning and argumentation**. Through these problems, debate and discussion are encouraged in groups.

The group's **research, discover, and present** different data, demonstrating **independence in judgments and arguments.**

16. In a restaurant are recorded data "On food preferences from some customers." The data are as in the table. Calculate the mode and median.

Menu numbers	1	2	3	4	5	6	7	8
The frequency	12	15	7	20	15	18	3	10

17. In a statistical study on the mass of 7 children, the smallest mass is 18kg, and the amplitude is 4. Mode,median,and arithmetic mean are 20 kg. Show that children can meet these conditions.

18. The table shows a certain number of people that are found in lunch in the restaurants of a city.

Number of people	8	12	16	17	18
Frequency	4	4	5	3	2

a) How many restaurants were surveyed?

b) How many people are counted in total?

c) Calculate the mode and median

d) Calculate the arithmetic mean

e) How many restaurants have more people than the median person per restaurant?

19. The table gives the number of notebooks that were in the children's bags on a typical school day.

Number of notebooks	Number of students
3	25
4	28
5	35
6	32
7	24

8	11

a) How many students were surveyed?

b) How many notebooks are counted in total?

c) Find out the mode.
d) Find out the median.
e) Find out the arithmetic mean
f) Find out the amplitude.

20. The pencils that were in some boxes in an office were counted and the data are in the table.

Number of pencils	10	15	20	25	30
Number of boxes	5	6	4	3	2

a) How many boxes have pencils more than the arithmetic mean?
b) How many boxes have more pencils than mode?
c) How many boxes do pencils have less than the median?

Exercises and problems 16-20 aim to enable students to find the average values whenthe collected data are given in density tables. The exercises have context from real life,and in this way students *touch*on the role of mathematics in everyday life, and understandthe importance of learning it.

21. (Gifted and talented). In a class of 25 students, girls are as much as 3/2 of boys. The average grade for girls is 8.5 and for boys, it is 7.5. Find the grade point average of the class.

22. (Gifted and talented). The arithmetic mean of the 2 numbers a and b is 22 and the number a is 4 times 3 times b. Find out the value of a.

The above exercises serve to differentiate the content, as students are gifted and talented, if not followed carefully, given the dose of difficulty they need, theexercises they work with can lose interest in mathematics.

1.6.3.5 Ratio and proportion (sixth and seventh week)

Ratio and proportion as part of the number theme in seventh-grade math aim to develop in students the following skills:

- Identifies the ratio as the quotient of two quantities.
- Writes the ratio in decimal form and percentage.
- Builds simple reports.
- Divides a size into two parts according to a given ratio.
- Connects the ratio with the proportion.
- Uses reports and proportions to solve problem situations from real life.

Thematic plan:

- Ratio
- Divide into a given ratio

- Exercises on the ratio (2 hours)
- Proportion (2 hours)
- Problems with ratio and proportions context-based from real life (2 hours)

The concept of ratio and proportion is revived in the 7th grade after a concise treatment was given to it in the 6th grade.

The smartphone in the first hour of this topic served to reflect each of the students, the mapping of concepts that they had realized with the knowledge they knew about the ratio.Illustrations with different figures were also created which illustrated the meaning of the ratio of the two quantities.

In the first lesson on this topic, the knowledge that the students recalled from the sixthgrade on the ratio was identified through the solution of exercises and problems 1. – 4.

Problems and exercises, and corresponding didactic comments:

Problems 1-4 were used to recall the knowledge about the report that students have received since the sixth grade.

1. The set $A = \{3, 5, 6, 7, 8, 9, 10, 11, 12, 15\}$ is given.
   a) What percentage are prime numbers?
   b) What percentage are composite numbers?
   c) What is the ratio of prime numbers to composite numbers?
   d) What is the ratio of composite numbers to prime numbers?

2.The classroom of VI A class has a width of 5 m and a length of 800 cm. Findout the ratio between a) width and length; b) length and width; c) express these ratios in decimal numbers.

3. What is the ratio between the lengths of the two sides of the rhombus?

4. The following is a recipe for a type of cake:

   100 g flour
   300 g of potato starch
   200 g butter
   200 g powdered sugar
   50 g of ordinary sugar
   5 egg yolks
   1 packet of yeast for cakes
   a little salt
   vanilla as desired.

   a) Write the ratio between the quantity of flour and starch.Simplify the ratio.
   b) Write the ratio between the mass of butter and regular sugar.Simplify the ratio.
   c) Ana has 200 g of potato starch and wants to use it all to prepare this type of cake
      - How much flour will she use?
      - How much butter will she use?
      - How much-powdered sugar will she use?
   d) Cindy says the ratio between eggs and flour is 1:20.Is she right?

Through updating the knowledge and various sketches on the SmartBoard, these conclusions were made.

- The ratio of two sizes *A* and *B* can be written *A* to *B*, *A* / *B*, or *A*:*B* (ordering is very important)
- The ratio shows how many times larger is the first size than the second when they are expressed in the same unit and are of the same type.
- The ratio can be expressed as a fraction, a decimal number, and as a percentage.
- If the value of a ratio is greater than 1, the first size is greater than the second size.
- If the value of a ratio is less than 1, the first size is less than the second size.
- If the value of a ratio is 1, then both numbers or quantities are equal.

6. Class VII B will plant 18 flowers in the schoolyard. Find how many flowers will be planted in grade VII C, if their ratio will be 0.6.
7. A new classroom will be equipped with 48 tables and chairs. Find the number of tables and chairs, if they stand in the ratio $\frac{5}{11}$.
8. In an orchard will be planted pear and apple seedlings in a ratio of 4:7. Apples were planted 81 seedlings more than pears. Find out how many seedlings were planted for each.
9. Determine the dimensions of a rectangle, if the ratio of length to width isand the perimeter is 25 cm. Find its surface.

Exercises 5. – 8. completed the framework of knowledge that students should possess on the report. Constructed through real-life situations, they reinforced the role of mathematics in everyday life and stimulated students' desire to solve them. Sketches with coloured parts on the SmartBoard brought a faster reaction to the procedure of their solution. These exercises develop competence inconceptual connection, mathematical thinking, and communication, mathematical reasoning and validation as well as problem-solving.

10. With 4 kg of milk is produced 1 kg of cheese. How many kg of milk is needed to produce 15 kg of cheese?
11. 4 m fabric for curtains cost 6600 ALL.
    a) How much will 1 m of this cloth cost?
    b) How much money will 10 m from this cloth cost?

12. Seven small melons of equal mass weigh 3010 g.
    a) Find the mass of a melon.
    b) Find the mass of 11 melons of the same mass
13. A washing machine at work consumes 4 kilowatts/hour of electricity for 2 hours. How many kilowatts/hour of electricity does it consume in 2.5 hours?
14. Andy is paid 25.27 euros for 3 working hours. How many euros will he be paid for 15 working hours?
15. An oil pump fills an 8000-litre tank in 40 minutes. How many hours does it take to fill a 12,000-liter tanker?
16. Posting 12 postcards cost 60 lALL. How much does it cost to send
    a) 8 postcards c) 10 postcards
    b) 15 postcards d) 18 postcards

17. The ticket for a show for 5 children costs 2250 ALL. How much do tickets cost?
    a) 4 children c) 6 children

b) 7 children d)9 children

18. 4.8 m rope cost 80 euros. How much money will we pay to buy the quantities of ropes below?

a) 3.6 m b) 6 m c) 9 m d) 12 m

19. Two groups of students went on an excursion to Saranda. The first group of 45 students went with the first bus and paid a total of 72,000 ALL. Each student paid the same amount of money. The second group of 34 students went by the second bus.Find how much money a total of 34 second-group students have to pay if the payment for each student is the same as that of each student in the first group.

20. Ingredients for 16 cakes are given below:

225 g butter, 225 g sugar, 75 g syrup, 275 g flour

Determine the mass of each of the ingredients to prepare:

a) 12 pieces b) 20 pieces c)24 pieces

Exercise group 10. – 20. reinforced the knowledge on proportion and brought a groupreal-life situations that required solutions through the concept of the report. Illustrations of solution schemes as well as mathematical reasoning for their solutions favour developing competence in problem-solving as well as thinking of mathematical communication.

The following problems are problems that were served to talented students. They require a higher degree of reasoning, and consequently the way of mathematical argumentation is finer and more detailed.

Exercise for gifted and talented students.

21. Two quantities are proportional to the numbers 5 and 7 and one is 24 units smaller than the other. Find how many units each of the quantities are.

22. To perform a job, 20 workers worked 15 days out of 8 hours a day. How many days, need 10 workers will perform the same work, working 12 hours a day?

23. Two sisters 10 and 15 years old will share the amount of 70,000 ALL concerning their ages. How much ALL belongs to each? But if they would divide this amount after 5 years, again about their ages, how much ALL would each receive?

24. The ratio of the lengths of the two segments is 2:3. The product of their lengths is 150 cm^2. Find the lengths of each. Is the number the ratio of the product of their lengths to the lengths of each of the segments? Argue why?

In gifted and talented problems, the connection of the concepts' ratio and proportion with the knowledge from all topics of mathematics is conceived to analyze in depth the problem situations by making diagrams and analyzing the connections they have between them confirming the uniqueness of mathematics.

1.6.3.6 Time measurement (Eightweeks)

Thematic plan:

The knowledge obtained in this chapter belongs to the topic of measurements. This topic comes through the knowledge of measuring time where they are treated:

- Time and its units of measurement
- Operations with time units
- Situations from everyday life where time units apply (2 hours).

Knowledge for measuring time is the knowledge that is obtained in the spiral cycle in each class but in the seventh grade, the level of difficulty of the exercises and concepts that are treated is a little higher.

Thus the place of exercises that have to do with the return of temporal units to each other is taken by exercises which require a higher level of reasoning. The questions posed there are related to the development of all competencies in the field and are mainly questions related to comprehension, where students use the information learned to solve problems. Various exercises and problems related to analysis and synthesis are present to support the development of mathematical competencies for all levels of students.

Problems and exercises, and corresponding didactic comments:

1. Find out :
   a) How many minutes are in 6 days?
   b) How many seconds are in 3 hours?
   c) How many days are from January 1st to August 31st (including these two)
      - In a leap year?
      - In a, not leap year?
   d) How many hours are there in 5 weeks?
   e) How many hours are in 14 400 seconds?
   f) How many minutes are in 3240 seconds?
   g) How many weeks are in 1176 hours?
   h) Which time is greater than 3 days, 4896 minutes or 254 880 seconds?

This exercise serves as a learning situation for the first lesson on this topic. Through it, students recall the relationship between units of time taken in sixth grade and create the basis for further treatment of actions with units of time in other exercises.

2. A bus leaves at 8:30 p.m. Has travelled 5h 40 min. What time did he get to the right place?

3. Joni is a teacher. He starts work at 8:45 a.m. and finishes work at 3:15 pm. He has a break from 11:10 a.m. to 11:25 p.m. Lunch break is from 12:30 pm to 1.05 pm
   a) How long is his work day (including vacations)?
   b) How many hours a day does he explain?
   c) The time to start work is 15 minutes before the start of the lesson and all teaching hours are on the equal time. He conducts 5 teaching hours. How long is a lesson?

4. Enkei and Jon are playing a game of tennis. They start the game at 10:35 and finish at 13:15. The game takes place in 4 sets. How long does a set last on average?

5. One clock lags behind 1min of the 30s per day. How far back will it be after 5d of 12h?

6. A wheel makes 42 turns in one minute. How many spins will it make in 1d 8 min 20s?

7. From one source flow 25 l of water per minute. How many hl of water will flow in 2h and 45 min?

8. To do a job it takes 15d 7h 50 min. If they are done 3/7of the job, how long does it take to complete the work?

Exercises 2. – 8. are exercises that require students to perform actions with units of time using the conceptual connection with other topics such as those of the number of operations with numbers of units of measurement of fluid volume. These exercises are slightly more advanced than the exercises set out in the text of mathematics and aim at developing the mathematical competence of conceptual connection and reasoning and mathematical thinking.

9.Convert speed to meters per second.

a) 80 m/min
b) 900 m/min
c) 120 m/min
d) 12 km/min

10.Convert speed to kilometres per hour.

a) 1 km/min
b) 0,2 km/min
c) 0,8 km/min
d) 3 m/s
e) 5 m/s
f) 10 m/s

11.Altin goes to work by bicycle. He leaves home at 07:45 and moves at an average speed of 15 km / h. The road from home to work is 5 km long. What time does he get to work?

Convert speed to kilometres of meters per hour.

a) 1 km/min
b) 0,2 km/min
c) 0,8 km/min
d) 3 m/s
e) 5 m/s
f) 10 m/s

12.Altin goes to work by bicycle. He leaves home at 07:45 and moves at an average speed of 15 km/h. The road from home to work is 5 km long. What time does he get to work?

13. Below is a part of a table for train movement schedules.

Stations	Departure time	Stations	Departure
Rrogozhinʌ	08:44	Elbasan	14:30
Peqin	09:01	Papʌr	15:00
Bishqem	09:17	Bishqem	15:31
Papʌr	09:49	Peqin	15:47
Elbasan	10:20	Rrogozhinʌ	16:04

a) How long does it take for the train departing from Rrogozhina to reach Paparr?

b) How long does the train that leaves at 08:44 from Rrogozhina to arrive in Elbasan?

c) Andi took the train at 09:01 from Peqin to Elbasan.

d) He spent 3,5 hours in an activity organized in Elbasan and took the afternoon train to return home to Peqin.

e) How long did he stay on the train on the way back?

Exercises 11., 12., and 13. are exercises which justify the connection of mathematics with other sciences, as well as with everyday life situations. They deal with the connection of time units with other units in composite quantities. Their level of difficulty is above average and requires **conceptual connections** and **mathematical argumentation**. So, in addition to problem-solving, other competencies are developed and all together play an important role in creating students' skills and abilities.

1.6.4 Personalized tests based on student levels.

At the end of the third term, a test was conducted to measure the level of student achievement for the third term. As part of student assessment, test design is of particular importance. The following steps were followed for its construction:

- The knowledge to be tested was determined
- The percentage that they would take in the test was determined based on the space of the topics in the thematic plan of the quarter and the level of difficulty of the questions.
- Specification table defined (Blueprint)
- The evaluation scheme was designed and the points were converted into grades in a table at the end of the test.
- The duration of the test was 60 minutes.

Below are two personalized tests for this quarter.

Test for the third quarter (Level I + II) Mathematics 7

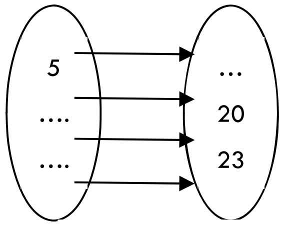
Take the test problems through all the stages of solving a problem.

1. In a factory that produces details, the probability of a defective detail coming out is 0.04.
How many details are expected to come out defective if the factory has produced 2000 details? (2 points)

2. In a box are inserted 6 red, 5 green, 3 yellow, 8 blue, and 3 black beads.
If we accidentally pull a bead out of the box, what is the probability that it is:
a)red b)green c) not yellow d) neither black nor red (2 points)

3. In a bag are found mint candy and strawberry candy. There are a total of 15 candies. The probability of getting a mint candy is $\frac{2}{5}$. If the first choice we make the candy is with strawberries, and we do not put it back in the bag, what is the probability that when, if we choose a second time, choose a peppermint? (3 points)

4. Fill in the domain and range for the following function. Argue. (2 points)



$y = 3x - 1$

5.Construct on the same coordinate grid the lines with the equation:$y = 3$, $y = 4 - x$.
What is the figure formed by their points and coordinate axes? (3 points)

6. A track transports 28kv of grain. Another transports 20% more. How many kg of grain do both tracks transport together? (2 points)

7.In a class 40% of the class are girls. The boys are 12. How many students are in the class? (2 points)

8. The price of a commodity was reduced from 300 ALL to 210 ALL. Find what percentage the price went down. (2 points)

9. The arithmetic mean of the three numbers is 42.6. Two of the numbers are 13.5 and 18.72. Find the next number. (2 points)

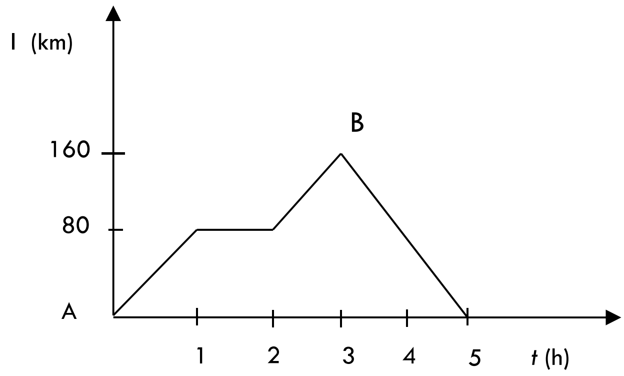
10. The age ratio of the two friends is 5: 7 and the sum of their ages are 24 years. Find out how old each one is and what their age ratio will be after 2 years. (2 points)

11. Ana and Era shared an amount of ALL in the ratio 3: 5. Ana received 250 ALL less than Era. How much money did the girls spend? How much money did each of them receive? (2 points)

12. A car does 524.4 km in 4 hours. Find out how many km it will take in 10 hours. (2 points)

13. A worker starts work at 08:30 and finishes it at 17:25. He also has 40 minutes break at lunch and 25 minutes in the afternoon. How many hours does he work in one day? What about in 5 days? (2 points)

14. The graph shows the route a car has taken from city A to city B and back. Answer the questions about the graph.



(4 points

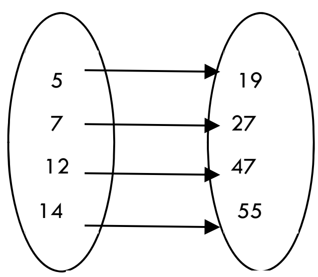
- How many hours did the trip last?
- How many hours did you arrive in city B?
- Did the car take a break on the road?
- How many hours did the return take?

The table of assessment.

Mark	4	5	6	7	8	9	10
Points	0-8	9-12	13-16	17-20	21-24	25 -28	29-32

Test for the third quarter Level III + IV Mathematics 7

1.In a factory that produces details, the probability of a defective detail coming out is 0.04.
How many details are expected to come out defective if the factory has produced 2000 details? (2 points)
2.Find the formula that connects the domain and range in the function represented by the arrow diagram. The plot in



$y =$ ....

the coordinate grid is the graph of the found function.(3 points)

3. In the coordinate grid construct the lines with equation $y = -2$ and $x = 3$. Construct in the same coordinate grid, two other lines so that the figure is square with an area of 25 square units. How many such lines can you build and why? Justify your answer. (4 points)

4. One box contains 20 small packets of 25.8 grams of baking soda each and 10 other packs of 32.4 grams of baking soda each. Find how many kg of baking soda there are in 15 such boxes. (2 points)

5. The age of 20 people in yearsare:

20, 22, 25, 31, 28, 23, 22, 26, 22, 27, 23, 23, 26, 28, 29, 30, 31, 21, 22, 27.

For the age years of these persons find:

a) Mode
b) Arithmetics mean
c) Median
d) Amplitude

(4 points)

7. Find the perimeter and surface of the shaded figure. The dimensions of the rectangle are 14.6 cm by 10.4 cm and the sides of the squares are 1.8 cm and 1.2 cm respectively. Find the percentage of the area of the rectangle,which is the area of the shaded figure. (5 points)

8. 4 students, 2 boys and 2 girls worked to complete a project. The two girls worked respectively, the first 4 hours, and the second 6 hours, while the two boys worked the first 5 hours and the second 8 hours.
Find out what part of the project working hours the girls did. What about the boys?
If 4600 All were paid for the project, find out how the boys and girls will share this amount of money in direct proportion to the work they have done. (4 points)

9. A man, after spending 50% of the money he had, spent as well $\frac{1}{3}$ of theirs, and he was left with another 600 ALL. Find out how much money this man had. (Make a sketch and reason). (3 points)

10. In a class 40% of the class are girls. Boys are 5 more than girls. How many students are in the class? How many boys and how many girls (4 points)

11. A movie starts at 20:30. At 20:33, 5 minutes of commercials were given and a quarter of the movie had passed. Find out at what time the movie ends, if the same commercials are rewritten twice. (3 points)

Table of assessment.

Mark	4	5	6	7	8	9	10
Points	0-8	9-13	14-17	18-22	23-26	27-30	31-34

1.6.5 *Learning/ teaching design implementation*

A very important process in problem-solving is the construction of the scheme of solutions illustrated with colours and sketches, looking for different ways of solving and passing from a specific concrete situation to the most abstract mathematical model, based on reflection and perceptions.

Throughout this period, SmartBoards were used to support this whole learning process, which was intended to be personalized and to serve everyone according to their abilities and needs. SmartBoard supported the illustration of exercises differentiated by levels of probability, fractions, ratios and percentages, processing of statistical data in seventh-grade mathematics

Although the final schemes were illustrated in SmartBoard, students worked in groups or pairs building schemes and justifying solutions as well as looking for new ways of solving them. Then in groups, they presented the solution to the problem on the SmartBoard illustrating in each case their actions. (Figure 43)

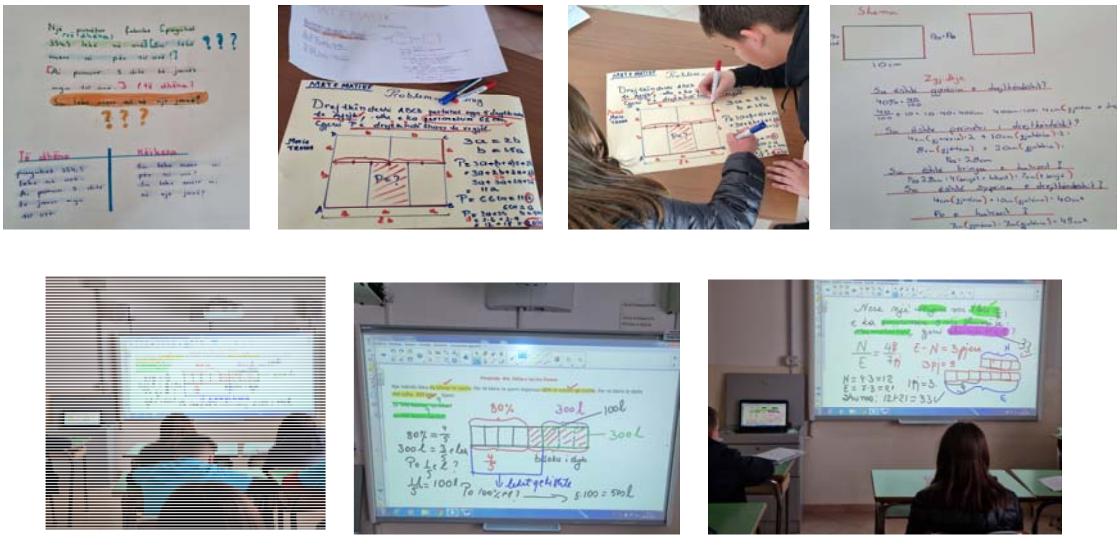


Figure 43Illustration of solutions made on paper and on the Smartboard

Using SmartBoard to illustrate solutions, increased the competition of groups of students to present the most beautiful solution for the exercise. Students were looking for different ways to make their solutions more interesting, and some of them even spent time after school reviewing the solutions to the problems illustrated in SmartBoard.

The math teachers of both groups were asked to apply in their classes also teaching through problem-solving.

In classes developed according to this active methodology they initially presented the problem and students were given about 5 – 8minutes to work independently at their own pace while the teacher monitored each student's approaches.

Subsequent discussions gave each of the students the opportunity, regardless of their level in mathematics, to discuss and share ideas and make comparisons about the ways of solutions given by others, enabling everyone to benefit from their own pace. and enrich their ideas with new ideas from those of their peers.

One problem that was noticed during the development of these special classes by both teachers was that in these classes one had to go beyond "problem-solving".

Anticipating different solutions, even the wrong ones, as well as their reasoning and discussion focused on the ultimate goal of the lesson, required thorough prior planning.

The teacher of the experimental group was favoured, as she had the opportunity to visually organize the ideas of the solutions more attractively so that they could be compared more easily.

She often brought up ready-made schemes in advance and used them to present to students, then asked them to formulate a problem that had that solution scheme.

Familiarizing students with the scheme, understanding the problem, and creating the problem that was supposed to have that solution scheme, automatically raised to a higher degree of critical and abstract thinking.

Also, the concentration of students in the experimental group hours was higher and students were more motivated.

1.6.6 Results obtained

The results obtained during the second quarter test compared to those of the first quarter are summarized in these tables. Important is the fact that in both groups, both control and experimental, thanks to the personalization of the learning process, either with traditional methodological tools or supported by the use of technology, there is an increase in the average grade of students in the test for the period. In the experimental group, this increase is + 0.44 greater, which shows that the use of SmartBoard, has affected the increase in student performance (Table 8&Table 9).

Table 8. The results of the control group

Class	Average grade (Second Quarter)	Average grade (Third Quarter)	Variation
Class 1 (20 studentes)	7,01	7,21	+0,2
Class 2 (19 studentes)	7,47	7,42	-0,05
Class 3 (20 studentes)	6,56	7,01	+0,45
The mean	7,01	7,21	0,20

Table 9 Results of the experimental group

Class	Average grade (Second Quarter)	Average grade (Third Quarter)	Variation
Class 1 (28 studentes)	8,17	8,74	+0,57
Class 2 (20 studentes)	7,10	7,63	+0,53

Class 3 (31 studentes)	8,44	9,26	+0,82
The mean	7,90	8,54	+0,64

1.6.7 Conclusions

Among the students of the experimental group, it was observed that differentiating the levels of the students through comprehensible schemes using graphs or different colours to reach them in a more comprehensible and visually appealing way, made them curious, growing in this way even the duration of their concentration.

The presentation of the material in an interactive way, in addition to the students 'performance, influenced the students' emotional behaviours and the approach they started to have towards mathematics after experimenting.

They began to show interest, and even the math teacher says she often found them in small groups reviewing solution schemes or other learning materials stored on the SmartBoard.

On the other hand, the math teacher of this group said that the time she had available was greater, as ready-made materials prepared at home could be downloaded directly to the desktop from her email and she could present them on the classroom SmartBoard.

In short, in addition to increasing student performance, there was also an increase in teacher performance.

The basic principles of personalized learning were rigorously followed in both groups. Attention was always paid to the development of key competencies which were specific to the topics developed during this quarter.

Also, the treatment of concepts and the construction of new knowledge, based on the problem and further the treatment of problem-solving according to the defined steps made it possible for the students to progress according to their pace by doing the simplest analyzes, for the students of a basic level, up to the most specific for upper-level students.They gave well-thought opinions and the quality of the judgment increased from week to week.With the increase in judgment,it came motivation and an increased desire to deal with mathematics.

Mathematics in our pre-university education is presented as a field of a dual nature.

1. On the one hand, through counting, measuring, modelling, and geometric concepts, students discover the world around them and provide the language and basic techniques for managing many aspects, including those of everyday life.
2. On the other hand, with the power of abstraction, logical argumentation, and the beauty of proof, it is presented as an intellectual discipline and as a source of aesthetic pleasure.

The strengthening of both of these characteristics is done in the first place by following the personal learning profile of the students, to design the technique of strengthening methods as well as having the support of ICTs tools.

2 The efficiency of dynamic GeoGebra software in offering differentiated instructions, based on students' levels: treatment concept of functions, in the eighth grade

2.1 Introduction

As the range of technological tools that facilitate daily activity grows, so does the interest of students to base their daily, most important, learning activities on technology. Learning mathematics as a very important process of student formation is under the constant positive influence of technological education tools that facilitate its understanding.

In all this very complex process, intelligence and fantasy are needed, **firstly** to choose the platform or technological tool with which to bring students closer to technology, and **secondly** to choose the way to use it foran efficient learning process.

Both of these points are very important to consider, despite the limited availability of schools regarding technological tools, or other restrictions of various kinds.

So, it is necessary to find a technological platform, or software that is provided for free and that allows students to access it also through mobile phones, and which we are competent to implement.

2.2 A brief overview of the role of GeoGebra in teaching/learning mathematics.

GeoGebra, developed in 2001 as part of Markus Hohenwarter's master's thesis, is today one of the most powerful applications that offer simultaneous technological support in the field of mathematics. It is an interactive application, which supports learning and explaining many areas of mathematics, such as geometry, algebra, statistics, and calculus. *GeoGebra is the right tool to support Math education if used wisely* (Wassie & Zergaw, 2019).

In the study "Integrating GeoGebra into space geometry in college" presented at the 3rd International Conference on Learning Innovation and Quality Education, it is said that: *This software should be introduced to students of mathematics teachers so that later can explore the world of mathematics more broadly and make students of prospective mathematics teachers able to think critically and creatively in learning geometry.* (Pamungkas, Rahmawati, & Dinara, February 2020).

The use of technological tools brings excellent opportunities to delve into the world of mathematics and discover its greatness.

Digital educational technology is not a*magic key* that will open the door to solving all the problems that exist in teaching and explaining mathematics, but it is a way to intervene with its potential in almost all areas: as in cases where the visual representation of concepts is required, such as the construction of figures, graphs, construction of graphs of functions, mathematical calculations in the field of measurements, presentation of figures in a geometric transformation, animated videos that explain different geometric transformations, and many other cases concerning to teaching e-learning in math.

Successfully planning and managing mathematical activities that will be supported by ICTs is the task of the mathematics teacher, who should plan where, when and with what t tool he/she will intervene towards the consolidation of a certain concept.

In 2009 researchers L. Fahlberg-Stojanovska, and V. Stojanovski, in the article "GeoGebra - freedom to explore and learn", showed *how the GeoGebra freeware (www.GeoGebra.org GeoGebraWiki: www.GeoGebra.org / wiki GeoGebraForum: www.GeoGebra.org/forum) – which gives the freedom to explore and learn to everyone, everywhere and at any time – can be of tremendous value to pupils and students in their understanding of mathematics from the smallest ages on up.* (Fahlberg-Stojanovska, Stojanovski, & Vitomir, June 2009)

And only 2 years later, in 2011, in the article "Building Simulators with GeoGebra" (Novak, Fahlberg-Stojanovska, & Renzo, 2011), researchers based on the teaching benefits that GeoGebra brings,*defined a holistic learning model where students and teachers join together in an adventure of learning mathematics by doing the mathematics.*

This chapter includes examples of simulators that are presented at various levels, starting from the end-user with polished ready-to-use simulators up to basic do-it-yourself construction problems, where students own the design by creating a simulator themselves.

GeoGebra's role in teaching mathematics takes on an extraordinary dimension when we see that various studies conducted in rural areas bring fantastic results regarding the use of this software.

Thus in their article "The use of GeoGebra in disadvantaged rural geometry classrooms", the authors studied the role that GeoGebra plays in the study of quadrilaterals in rural schools. The conclusions they bring are very satisfying: *Results from the quantitative data analysis showed a significant difference in the mean scores with a mean difference of 6.5 in favour of learners taught with GeoGebra compared to the chalk-and-talk method. Implementation of and enthusiasm forGeoGebra had a positive influence on learning achievement. Based on the results, it was concluded that GeoGebra as a pedagogical tool can work effectively in deep rural schools where geometry is hardly taught.* (Manganyana, Putten, & Rauscher, 2020).

In the range of studies, related to the integration of technology in teaching, what is noticed is that most of them shed light mainly on the integration of GeoGebra in the field of geometry, although many other studies shed light on its integration in algebra, calculus, and trigonometry. They bring scientific facts and details about the reasons why this software manages to strongly influence the performance of students, their logical and critical thinking, and the way they go from general to abstract, passing from mathematical learning to doing the math.

On the other hand, *In the modern era where technology usage is a tradition of the generation, integrating teaching and learning with mediums that could catch up and satisfy pupils' interest is noteworthy* (Wassie & Zergaw, 2019).

In the same study, the authors see GeoGebra, as software with great contributions to fostering students' interests and achievements, and as an environment to flourish different learning styles.

Another interesting study on the use of GeoGebra comes from Grace Ramatsimele Kekana in his article "Using GeoGebra in transformation geometry: an investigation based on the Van Hiele model.", which sheds light on its use in understanding geometric transformations by lower secondary school students. In his article, the authors state *that students at this level can express generalizations and informal arguments about what they have previously learned to validate the rules drawn. Also, students at this level were able to reach a level of abstraction* (Kekana, 2016).

The efficient use of this dynamic software is closely related to the professionalism of teachers, the way they implement the software in teaching, the purpose they have when implementing it in teaching, and what they should do in advance to personalize the learning.

With GeoGebra introduced in math teaching, the teaching/learning process is not only improved in terms of speed and quality. Mathematical concepts, rules, and procedures must be adapted to the new environment. On the other hand, the characteristics of the computer and educational software in use must be thoroughly examined and a successful teacher must use their advantages and avoid their disadvantages to maximize the efficiency of the teaching process. (Ljajko & Ibro, 2013)

2.3 Case study on the efficiency of dynamic GeoGebrasoftware, in giving differentiated instructions based on student levels, on treatment concept of functions, in eighth grade

Dimensional stages of personalized learning always mean that in addition to personalized instruction for each of the students' levels, digital support educational technology is also chosen to make maths learning efficient and engaging at the same time.

After training on learning styles, personalized learning, ICT support to adapt teaching to students' learning styles and assistive technology tools, the use of the dynamic GeoGebra software was applied to support the personalization of learning in mathematics, in a secondary school in the city of Elbasan.

This experiment shows the benefits of using GeoGebra software for the study of linear functions in the eighth grade of lower secondary education in Albania, focusing on personalized learning, supported by this dynamic software.

In this comparative experimental study, the efficiency of teaching through dynamic software such as GeoGebra was studied, providing differentiated instructions, based on student cognitive levels. The context is the topic bout functions in the eighth grade.

The choice of the topic 'Functions' for the eighth grade was done deliberately, as it will not only serve to develop a knowledge base that will be expanded in the 9-the grade, but the concept of the functions is more important for students, so they to be able to model and interpret solutions, problems from real life and other fields of science – chemistry, physics, biology, etc.

2.3.1 Characteristics of the study

Methodology: The eighth-grade students of two 9-year schools in the city of Elbasan were studied – students of the school "Qamil Guranjaku" (as a control group) and students of the eighth-grade of the school "Jorgji Dilo" (as an experimental group), and the treatment of knowledge about functions in both groups in the third quarter of the school year 2020-2021 was followed.

The experimental group (72 students) acquired the knowledge defined in the 8th-grade curriculum, assisted by the use of dynamic software. The control group (70 students) conducted the learning process in a regular classroom, according to techniques and methods fully defined, but not supported by digital technology.

The selected schools are in very close positions with each other in the ranklist of the schools' performance, based on the school's Performance Card.*(The Performance Card includes a set of indicators, the measurement of which provides information about the school results. These assessments* *are annual and are used for comparison and ranking at the regional level. The completion of the Performance Card is done by the school based on sources such as static data provided by the school, data from tests at the regional level, data from exams at the national level, data from questionnaires, developed at the regional level, data from questionnaires developed with students, parents, and teachers.)*

The school where the experiment took place has a rich computer lab and a SmartBoard. Math classes were held in the computer lab to use both SmartBoards and computers.

The textbook used: Mathematics 8. (Pemberton, Kivlin, & Winters, Essential Mathematics for Cambridge Secondary 1 stage 8 Pupil book, 2017)

Topic: Functions.

Measurable variable: Learning outcomes, represented by grades at the end of the topic learning in both groups and their approaches to model real-life problems through knowledge of functions.

Although they observed many aspects of teaching, the presented studyis limited to analyzing the data collected for only one variable as well as to the motivation of the students who underwent the experiment.

To monitor the effect that the use of GeoGebra brings on the personalization of mathematics learning on the topic"Functions" in the eighth grade, a pre-test and a post-test model were used to compare the level of the participating in the study classes and to measure the changes that would occur as a result of applying this dynamic software to personalize math learning. With the classes where the experiment was carried out, a pre-test was made concerning the knowledge of the function obtained in the previous classes, which was possessed by the eighth-grade students.

The results of the pre-test served as the initial database for the students of both groups regarding the mathematical skills and competencies that they possessed until the moment when the experiment started.

2.3.2 *Teaching/learning design*

Table 10contains the knowledge for the realization of mathematical competencies as well as the skills for the realization of these competencies that should be mastered by the students at the end of the seventh grade.

Table 10. The knowledge for the realization of mathematical competencies in the seventh grade

Class	Knowledge for the realization of mathematical competencies	Skills for the realization of the competencies of the field/subject
VII	• The pairing of elements of two sets. • Sorted pairs and coordinates. Functions. • Tabular and diagram way of presentation.	• Demonstrates intuitive understanding of functions. • Introduces simple functions with words, formulas, tables, and diagrams. • Constructs the graph of the function $y = ax$

	• Representation of functions with formulas. • Function graph.	• Constructs the graph of the line parallel to the x-axis or the y-axis. • Pair the elements of two sets

The realization of the pre-test for the knowledge about the function possessed by the students up to that moment had two essential reasons:

- To create a clear picture of the strengths, and weaknesses that the students of both groups had concerning the topic of the function, to further personalize the whole process, with or without the support of ICT on it.
- To compare the results of the two groups with the post-test,the test that is developed concerning the knowledge of functions obtained in the eighth grade.

At the end of the experiment, a post-test was performed, to investigate the extent to which the use of this software, in the treatment of the concept of function, influenced the increase in students' performance.

In the competency-based curriculum,https://ascap.edu.al/ in the mathematics program for the eighth grade, concerning functions, the knowledge and skills components of mathematical competencies are presented in theTable 11.

Table 11 The knowledge and skills forming mathematical competencies in the eighth grade

Class	**Knowledge for the realization of mathematical competencies**	**Skills for the realization of the competencies of the field/subject**
VIII	• Linear functions ($y = ax + b$). • The equation of the line $y = ax + b$.	• Expressing simple linear functions and presenting them with diagrams ($y = ax + b$). • Constructing value tables and using coordinates to construct their graph. • Recognizing that the equation $y = ax + b$ expresses a graph of a linear function.

Based on the results of the students in the pretest, in both groups, the knowledge for the realization of mathematical competencies as well as the skills for the realization of the field competencies for the eighth grade, a detailed plan was drafted containing a list of learning outcomes that should master the students according to the levels.

The expected learning outcomes, corresponding to 3 levels of achievements are:

The first level(base level)

- Distinguishes whether the pairing of two finite sets, given by an arrow diagram or a table, is a function.
- Writes ordered pairs (face-image), for a finite function given by an arrow diagram or table.

- Finds the value of a function given by a very simple formula, for a simple value of the variable and constructs the corresponding point in the graph.
- Detects if a point with given coordinates is located on the graph of the function $y = ax$ or $y = ax + b$.
- Finds the value of the function for a given value of the abscissa, when its graph is given.
- Constructs the graph of the function $y = ax$, for given values of **a**, using the fact that it passes through the coordinate origin.
- Constructs the graph of the function $y = ax + b$, defining in advance the coordinates of its intersection with the x-axis and that with the ordinate.
- Finds the angular coefficients of lines when their equation is given.

The second level(medium level)

- Distinguishes whether a given set of points in the coordinate grid serves as a graph of a function.
- Finds the set of definitions and the set of values, for a finite function given graphically.
- Finds the formula of the function ($y = ax + b$), when the table of its values is given.
- Distinguishes the sign of the slope of a line which is the graphical representation of a linear function (even when it is zero).
- Finds the points of the intersection with the coordinate axes of the graph of the function $y = ax + b$.
- Finds the slope of a straight line when its graphical representation is given.
- Finds the point of intersection with the axis of the ordinates for a graphically represented line, and makes the relation of the point ordinate with the value of b in the equation $y = ax + b$
- Writes the equation of the line when its graphic representation is given.
- Presents differently a linear function, given in words or letters.
- Solves simple problems that are modelled by the functions $y = kx$ or $y = ax + b$.

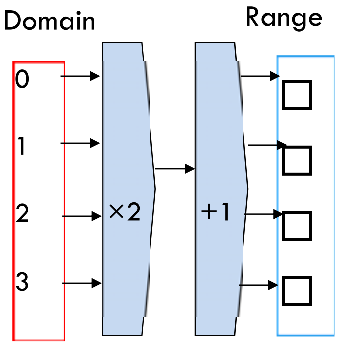
The Third Level

- Writes the formula of linear functions given in other ways.
- Writes the equation of a line which is the graphical representation of a linear function, when it is represented graphically.
- Writes the equation of a line, symmetric to a given line,about the ordinate axis, when it is represented graphically.
- Solve problems presenting real-life situations, mathematically modelled by functions $y = ax$ and $y = ax + b$.

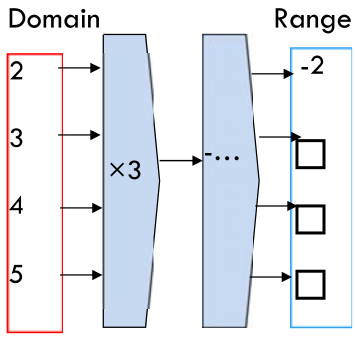
Initially, both groups underwent this pre-test, which tested the knowledge possessed by seventh-grade students on the function.

Pre-test onthe function topic

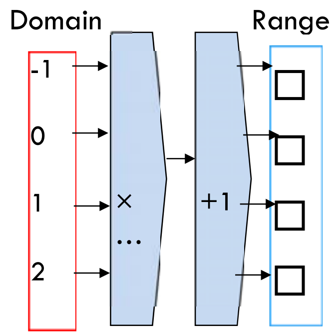
1.Complete the following diagrams.



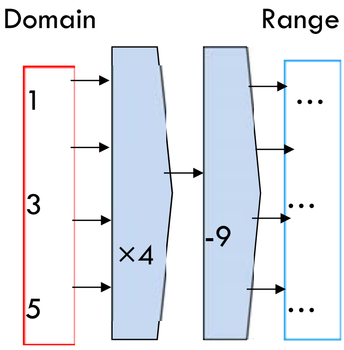
$y = 2x + 1$



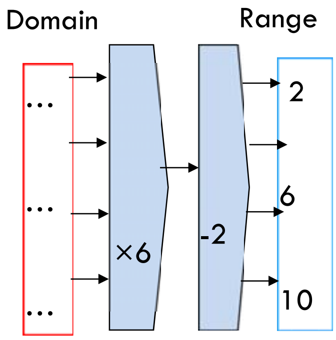
$y = \dots$



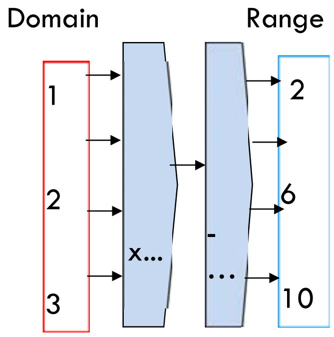
$y = 3x + 1$



$y = \dots\dots\dots$



$y = \dots\dots\dots$



$y = \dots\dots\dots$

(12 points)

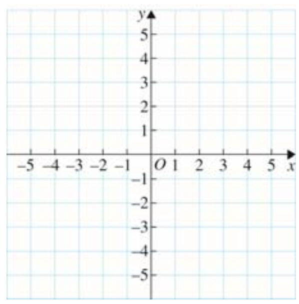
2.

a) Plot the lines, corresponding to the given equations:

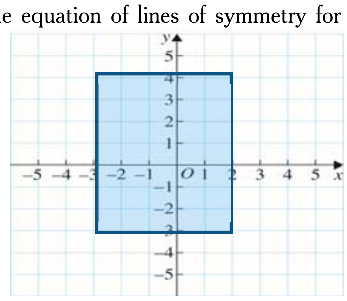
$y = -4$ $\quad x = 2$

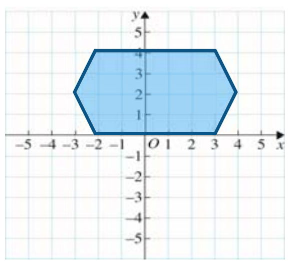
b) Construct two lines, parallel to the given lines, and also write their equations.

c) How many lines of equations can you write? Why? (6 points)



3. Write the equation of lines of symmetry for the following figures:(4 points)

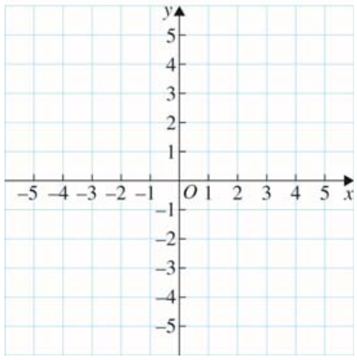




4. Write the coordinates of the three points that lie in a straight line. (1 point)

A(……;……) B(……;……) C(……;……)

5.For the functions given by the formula, fill in the table and construct their graphcon the coordinate grid. (4 points).



X	-3	1	2
y=x+2			

X	0	2	4
y = 2x–1			

6. Which of these points lie on the graph of the line, presented by equation $y = -3x + 2$? Why? (3 points)

A(1;4) B(2;7) C(3;-7)

Table of assessment:

Marks	4	5	6	7	8	9	10
Points	0-7	8 -11	12-15	16-19	20-23	24-27	28-30

The results of the pretest are presented in Table 12.

Table 12 Grade distribution table for frequencies and percentages for both groups in the pre-test

	Experimental group		Control group		
Marks	Frequency	Percentage (%)	Marks	Frequency	Percentage(%)
4	6	8,3	4	4	5,7
5	9	12,5	5	10	14,3
6	12	16,7	6	5	7,1
7	9	12,5	7	12	17,2
8	18	25,0	8	15	21,4

9	9	12,5	9	16	22,9
10	9	12,5	10	8	11,4
	SUM =72			SUM = 70	

In the table of the distribution of grades of the preliminary test, it is seen that in the experimental group there are 37.5% of students in the intermediate level (marks 7, 8), compared to 38.6 in the control group, whereas 25% of the students in the experimental group belong upper level (marks 9, 10), compared to 34.3% in the control group.

So there is a slightly higher level of math achievement in the control group. Refer to the average values of both groups can conclude that:

- For the experimental group: Median= 7.5, Mode = 8, arithmetic mean: 7.04
- For the control group: Median = 8, Mode = 9, arithmetic mean: 7.48 (Figure 44 Distribution of pre-test marks)

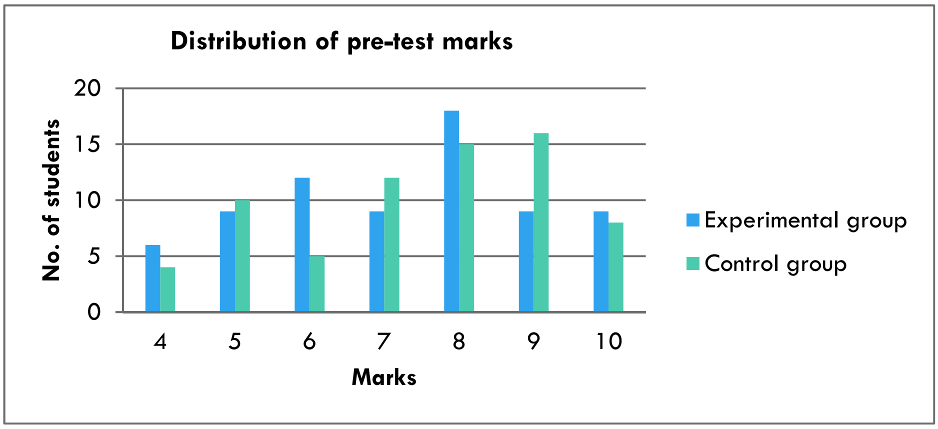


Figure 44 Distribution of pre-test marks

2.3.3 *Teaching/learning process implementations*

Based on detailed planning,corresponding to the class levels, the teachers of both groups organized lessons with each of the groups that offered opportunities for all levels of students, according to their abilities and needs. The teacher of the control group, provided the necessary teaching tools to enable qualitative development for the students in the control group, providing millimetre papers for graphic constructions, coloured pencils, as well as posters with coordination and graphic grids, built into them, prepared in advance.

On the other hand, the teacher, who worked in the experimental group, was more comfortable in her work, although the initial procedure of building graphs.In this group, the software usage was preceded by the construction of graphs on millimetric paper by pencil. The goal was for studentsto understand the algorithm of the whole procedure, and then to continue with the software to delve into high-level exercises and problems.

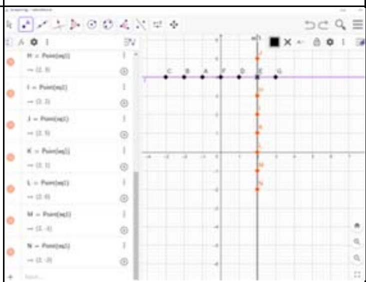
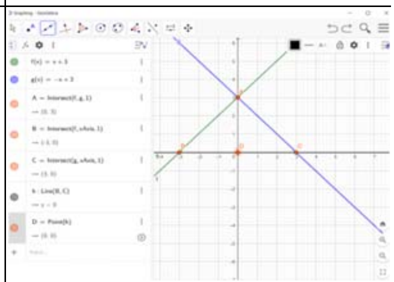
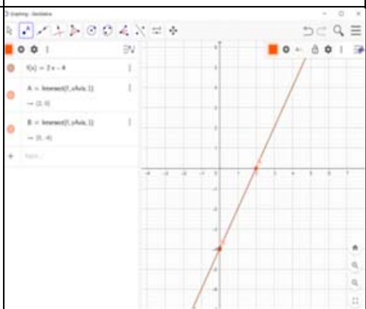
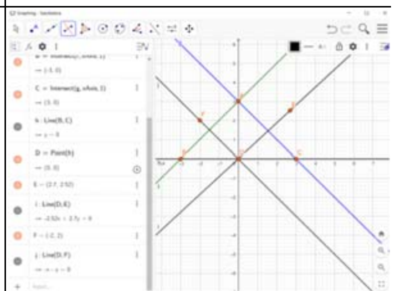
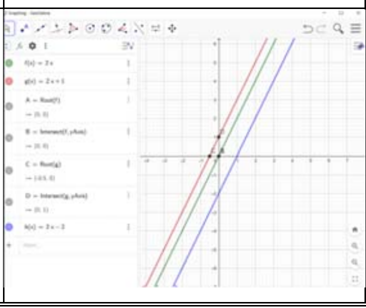
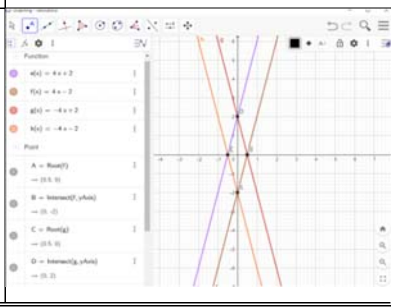
The learning activities in the control group took almost twice as long as the activities in the experimental group. Students in the control group were not focused as long as those in the experimental group.Visualization significantly increased students' interest in concentrating and participating actively in the process.

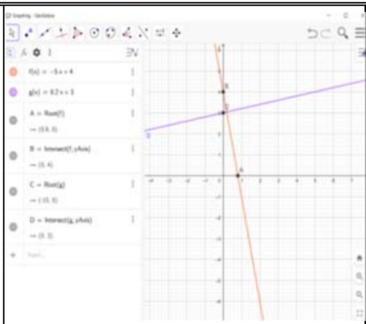
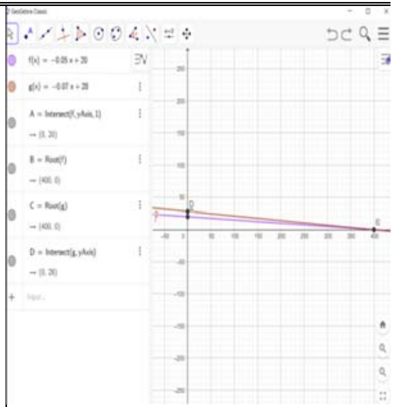
The number of exercises with a high level of difficulty, solved by gifted students in the same class was greater than that of the experimental group.

2.3.3.1 Activities, supported by GeoGebra usage

Table 13 shows some of the activities performed by students in the experimental group, based on their pace, needs and abilities.

Table 13. The activities enabled by GeoGebra

The activity enabled by GeoGebra	Graphs	The activity enabled by GeoGebra	Graphs
Graph of lines parallel to the coordinate axes		Investigation. Construct the graph of the symmetry of a line concerning an axis of ordinates!	
Construct the graph of the function $y = ax + b$		Investigation. What is the type of shape, formed by the intersection of the lines? Investigate the properties of this figure.	
Investigation on parallel lines. What is the relation with the slope coefficients?		Investigation. Find the area of the shape formed by the intersection of the four straight lines.	

Investigation on perpendicular lines. What is the relation with slope coefficients?		Mathematical modelling. A car has 28 litres of oil in its tank when it starts moving. Another car has 20 litres of oil. The first consumes 0.07 litres of oil per km, while the second 0.05 l/km.Enter the number of litres of oil found in the car tank after **x** kilometres of road. Model the function that connects y to x and interpret the function you modelled.	

Mathematical modelling is one of the six mathematical competencies that are developed through mathematical topics. The student describes and creates models using mathematical knowledge by presenting real-life situations with mathematical language. Further, using mathematical knowledge and appropriate techniques, a mathematical solution is found which is then interpreted in real life.

Modelling goes passes through these steps:

- The situation in real life is determined.
- It is then modelled in mathematical language.
- The mathematical solution is found.
- Interpretation of mathematical solutions in solving a real-life situation.

The same situation from real life was posed in both groups.

> A car has 28 litres of oil in its tank when it starts moving. Another has 20 litres of oil. The first consumes 0.07 litres of oil per km, while the second 0.05 litra/per km. Enter the number of litres of oil found in the car tank after x kilometres of road. Model the function that connects y to x and interpret the function you created.

This problem situation develops mathematical modelling. The students of both groups had a special interest in it, but what favoured the students of the experimental group was the accurate construction of the graph by the GeoGebra software and therefore, it shortened the time for more detailed study and interpretation by delving into the problem.

In the control group, many of the students had problems with the divisions on the graph paper in the construction of the two coordinate axes, and therefore the "time" factor conditioned them to advance by delving into logical reasoning and mathematical modelling of situations.

Some other problems solved by the gifted and talented students are presented below:

2.3.3.2 Differentiated tasks for gifted and talented students.

1. A student has to solve 30 exercises. He solves every day 2 exercises. How many exercises (s) are left after t days? Write a formula that relates the number of exercises n, to the number of days t. Draw the graph. Could point with abscissa 7 lie on this graph? Explain why.

2. A car at the moment of departure has in its tank 20 litres of diesel. It consumes 5 litres of diesel per hour of travel.
   a) Formulate the S amount of gasoline remaining in its tank 1 hour after departure.
   b) Graph the function that expresses the dependence of S of t (travel time). Show on the graph how many litres of gasoline there is in the tank 2 hours after departure, 3 hours after departure, and 4 hours after departure.
   c) After how many hours of travel in the car tank will have: 15 litres; 10 litres; 5 litres; or 0 litres?
3. It is known that the graph of the function $y = 3x + c$ passes through the point (-2; 5). Find the value of c and construct the graph.

4. It is known that the graph of the function $y = mx - 5$ passes through point $A(-3; -2)$. Find the value of the coefficient m.

5. Construct on the same coordinate grid the lines with equation: $y = 3x - 6$, $y = -3x - 6$, $y = 3x + 6$, $y = -3x - 6$. Mark their intersection points A, B, C, and D. Show the properties of the quadrilateral $ABCD$ and find its surface in square units.

6. A line intersects the y-axis at point $P(0; -4)$ and passes through point $A(1; -1)$. Write the equation of the line.
7. Construct the line with equation $y = -x + 5$. Write the equation and construct the line, which is its symmetrical lineconcerning the y-axis. Construct the symmetries about the x-axis of the two straight lines you obtained. Write their equations. What do you notice?

8. Construct in the same coordinate grid the lines with equations: $y = -2x$; $y = -2x + 1$; $y = -2x - 3$; What do you notice? What can you say about their mutual position? What do the equations of these lines have in common? Conclude.

9. Construct in the same coordinate grid the lines with equations: $y = -2x - 2$; $y = 0{,}5x + 1$. Measure the angle formed at the cutting point of these two lines. What do you notice? What can you say about their mutual position? What do the equations of these lines have in common? Conclude.

10. 2 of the parallel sides of a square lie on the lines with equation $x = -3$ and $x = 4$. Write the equations of the other two lines, where the other two parallel sides are located. How many lines can you write? Describe 3 cases.

Talented students have some special characteristics, that make them stand out from all others for the sharpness with which they cope with given situations. They mostly have the following characteristics:

- They are extremely sharp and show a constant curiosity about the world of numbers.
- Learn very quickly and just as quickly understand and apply newly weighted mathematical concepts.

- Think and work abstractly with high accuracy.
- They always have original ideas to work with, outside of the stereotypical way of solving a certain problem.
- They impress you with the way they transfer learning to new mathematical situations very comfortably.
- They perceive, visualize and generalize patterns and numerical and non-numerical relationships.
- The reason is quickly by analyzing, deducing, and inducing, according to the conditions of the problem that requires a solution.
- They formulate mathematical questions that apply concepts, related to them directly or indirectly.
- They do not give up on their quest to solve complex, non-standardized tasks.
- They can organize in-depth the information provided while simultaneously scratching off unnecessary information, and quickly discover hidden information to solve multiple-choice problems.
- They possess techniques of well-thought actions and often impress with the speed of their performance.

Based on the above characteristics, technological tools come as a golden opportunity to enable them to progress rapidly and by supporting and shortening the time they would likely lose without technology in routine actions, which do not enable logical and critical reasoning,advancing further from the concrete to the abstract as the highest form of reasoning.

At the end of the function module, the students in both groups underwent a control test that aimed to assess the level of achievement of both groups concerning the function chapter.

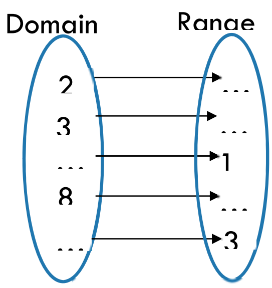
On the other hand, this evaluation was used to analyze the efficiency of using GeoGebra software in the experimental group.

The test was in full coherence with the purpose for which it was designed.

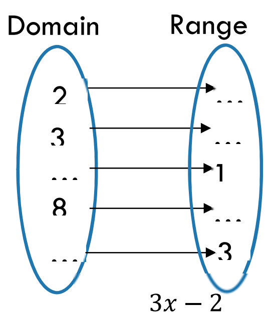
The test contained questions of all difficulty levels, which were placed on a scaled test. Care was taken with the taxonomy of the test. The distribution of points according to the three taxonomic levels was done in the ratios: 40%: 36%: and 24%.

2.3.3.3 Post-test on the "Function" topic

1. Complete the following diagrams. (5 points)

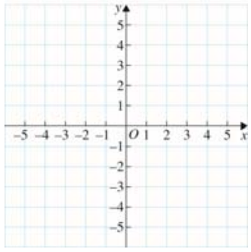


$y = 3x - 1$



$y = \frac{3x - 2}{4}$

2.Write the formula for the given function with the "machine" function. Fill in the table of values and plot the graph of the function. (3 points)



x → x 3 → +2 → y

Formula: $y =$ …………

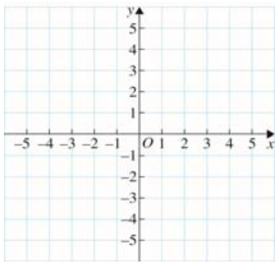
x	-2	-1	0	1
y				

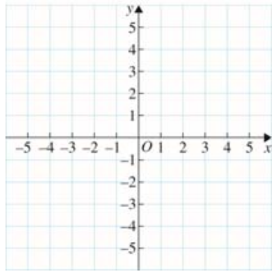
3. Plot in the same coordinate system, each of the pairs of functions given by formulas.

a) $y = 2x$ and $y = -\frac{1}{2}x$

b) $y = -3x + 1$and$y = -3x$

Write what you notice. (5 points)



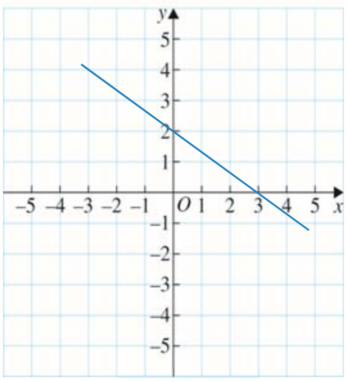


4. Write the formula of the function presented in the tables. (4 points)

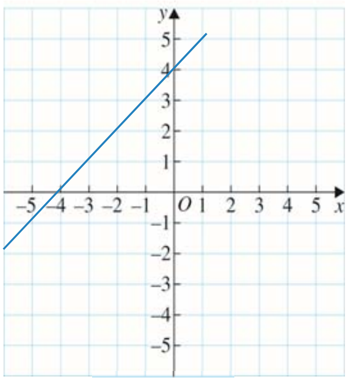
X	-1	0	1	2
y = …..	-4	-1	2	5

X	1	3	5	8
y =…	-3	1	5	11

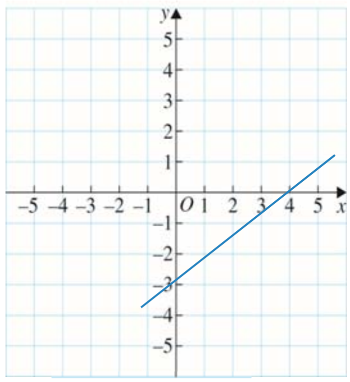
4. Write the equations of the graphs presented graphically. (6 points)



y……….



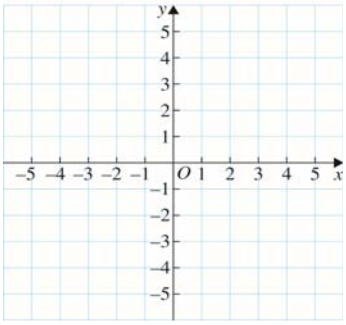
$y =$ ……



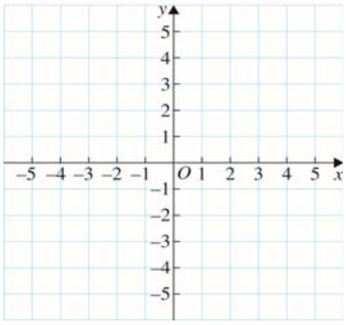
$y =$ ………

6. Plot on the coordinate system any graph of a function with the given slope and write the equation of a line: (3 points)

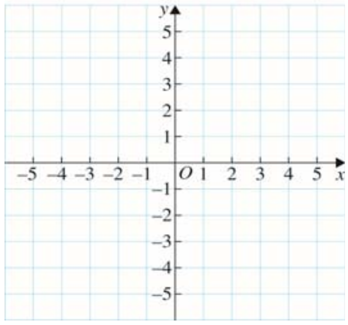
a) Negative



b)Positive



c)Zero



7. In a warehouse are 20 t of flour. The warehouse is supplied daily with 5 t of flour. With y,indicate the amount of flour in stock after x days. Write a formula that relates the y quantity of flour in stock to the number of days x. Graph the function. The storage capacity is 120 tons. After how many days will the warehouse be filled? Make a graphic interpretation of the solution. (4 points)

Achievement evaluation table:

Marks	4	5	6	7	8	9	10
Points	0-7	8 -11	12-15	16-19	20-23	24-27	28-30

2.3.3.3 Results obtained

The analysis of the final test for the function chapter showed that the group that underwent the experiment achieved higher results than the control group. Table 14reflects these results.

Table 14Distribution table for frequencies and percentages for both groups

	Experimental	group	Control	group	
Marks	Frequency	Percentage (%)	Marks	Frequency	Percentage(%)
4	2	2,9	4	5	7,2
5	4	5,6	5	7	10
6	7	9,7	6	8	11,4
7	15	20,8	7	16	22,9
8	15	20,8	8	12	17,1
9	17	23,6	9	13	18,6
10	12	16,6	10	9	12,8
	Sum =72			Sum = 70	

It is seen that in the experimental group there are 41.6% of intermediate levelsof students (marks 7, 8), compared to 40% in the control group. At the same time, 40.2% of the students in the experimental group belong to the upper level (marks 9, 10), compared to 29.4 who are in the control group.

It is noticed that the experimental group has a larger number of students at the upper level, while in terms of the intermediate level (marks 7, 8) there is no major difference between the two groups.

Referring to the mean values of the two groups for the post-test, the results are:

- Experimental group: median = 8, mode = 9, arithmetic mean: 7.88
- Control group: median = 7, mode = 7, arithmetic mean: 7.4

The graph in Figure 45shows the distribution of the marks of the post-test.

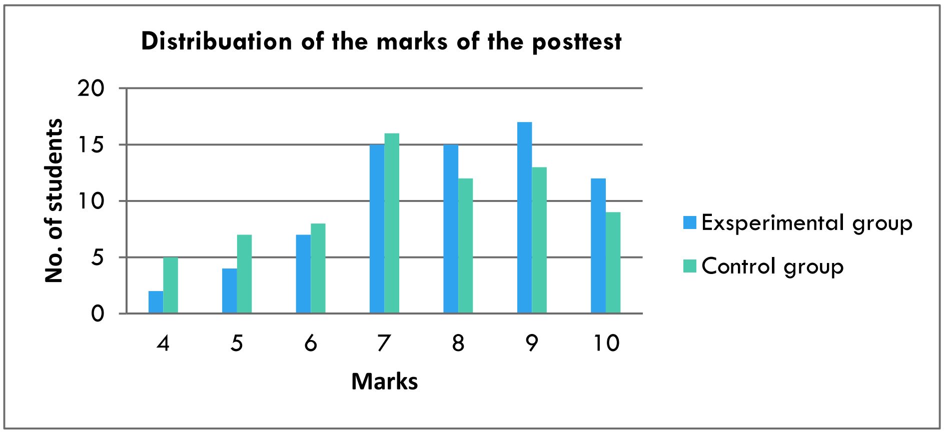


Figure 45Distribution of the marks of the posttest

In the graph,in Figure 46a comparative presentation of both groups in both tests is made.

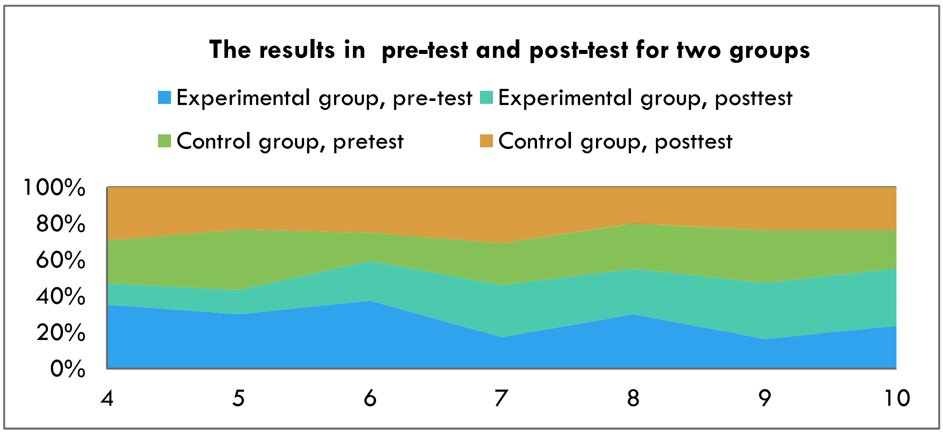


Figure 46The results in pre-test and post-test for two groups

In the experimental group, the area of the intermediate and upper-level marks in the post-test is larger than that in the pre-test, while we also have a narrowing of the area of the first-level grades, as well as non-passing grades.

However, for the areas that represent the control group, the same does not happen in the graph that presents the results of the control group.

2.4 Conclusions and recomandations

1. The post-test results in the experimental group increased by 0.84 based on the arithmetic mean, compared to the pre-test, while in the control group the arithmetic mean experienced a slight decrease of 0.08 compared to the pre-test.
2. The motivation and enthusiasm of the students of the experimental group, while learning mathematics, increased significantly.
3. The use of the program, based on student stories, was extended in the afternoon at home on their laptops or tablets. GeoGebra thus expanded the boundaries of learning, moving from formal to non-formal classroom education in their own homes and leisure discussions.
4. Increased student collaboration, not only within the classroom but also between classes helping each other using the app andenthusiastically explaining everything they learned through GeoGebra.
5. The students also benefited a lot in the field of ICT, greatly facilitating the work of the mathematics teacher, to personalize the teaching.
6. Using software shortened the time you would waste with graphic and sketch constructions, giving students the advantage of engaging in understanding and modelling as key subject competencies.
7. Using GeoGebra software was an experience that professionally enhanced even the teacher, who during all this time was practised in advance at home to bring out the best she could for the students.

On the other hand, the use of GeoGebra as a technological teaching tool to support personalized learning can be used to extend to all classes and topics, both in geometry and geometric transformations and in other topics.

Although this software helped to visualize the linear function, it is very important to maintain a balance between lessons, where educational technology tools can be used, and those, where students had to practice manually.

The practice of using GeoGebra in teaching requires training for students on how the software works and the areas of its use. It is recommended that in the subjects of informatics, in parallel with the subject curriculum, there are special topics, with instructions, for the way of using didactic software for learning.

This would increase students' autonomy to learn, as well as enable them to work with technological educational applications and software even in their spare time at home.

It would be ideal that the time students spend with the with their computers, phones or tablets to make it part of non-formal math education.

3 THE IMPACT OF LEARNING GAMES ON THE ATTITUDE OF SECOND-GRADE STUDENTS TO THE SUBJECT OF MATHEMATICS AND THE DEVELOPMENT OF STUDENTS' COMPUTATIONAL THINKING

3.1 Introduction

The world is rapidly changing its approach to technology. Our students as part of this change spend a lot of time in front of technological devices. The policies of education systems around the world are being structured in such a way as to implement a range of technology-based teaching methods so that learning is a process pleasing to the younger generation. When this process is supported by technological devices that students enjoy and spend a very long time with them, the most important thing to do is to orient and support so that the time spent with these technological devices favours their education and training.

In this situation, teachers have to provide technological support to increase students' creativity and ensure involvement in the learning process with desire and passion.

It is understandable that the long time that students spend with ICT tools cannot be avoided, but they can be offered alternatives that support the personalized learning process through games.

This experiment deals with the advantages of mathematical games in general and Kahoot! in particular, to motivate increase the self-esteem, and collaboration of students of different levels and strengthen positive attitudes towards learning mathematics.

The experiment confirms the role that learning games play in mathematics to develop logical and critical thinking, but especially computational thinking.

Throughout life, everything we learn willingly and that remains with us forever is based on curiosity and play.

When curiosity and play come together in a fun, stimulating, social form, the strength and potential to learn increases in each person, regardless of age or ability, but in particular in each child.

In the learning process in general, as well as in mathematics in particular, math games play a key role.According to (Oldfield, 1991)*Mathematical games are "activities" which: involve challenges against one or more opponents and are developed based on a set of rules, usually against one or more opponents and have a clear basic structure as well as specific mathematical cognitive objectives.*

3.2 A brief overview of the role that learning games play in the development of logical and mathematical thinking

Math games increase students' performance in the subject of mathematics. A very important aspect of math games is also the satisfaction and positive attitude that students feel during the development of the game. The need to motivate every student to learn math by fostering math discussion is one of the benefits that come from math games.

Regarding positive attitudes towards mathematics I want to quote (Obioma, 1992)who says that *the roles of Mathematical Games in the classroom are making practice periods more pleasant and successful, enrichment of vocabulary, introducing new ideas, allowing for individual differences, improvement of study habits, developing positive attitudes towards Mathematics.*

All math games play an important role in the development of students' many skills during math learning, but technological math games bring even more benefits. One of these games is Kahoot! application.

Kahoot! platform, is a game-based learning platform, used to assess students' knowledge, as well as to see the effect of moving away from the monotony of learning activities developed mainly in the classroom.

Mathematics teachers aim to promote students' logical and critical thinking through the application of various teaching methods and the use of various computer software. On the other hand, it is a kind of thinking, that no one better than mathematics as a field can develop. It is about computational thinking.

Kahoot app! is an application that is constantly updated and updated, day by day, aimed at empowering students in terms of learning mathematics, orienting them to move from just students to creating leaders of their interests related to learners.

The application also enables the exchange of ideas and interests between them, serving mutual growth based on interpersonal relationships.

Through the use of this application, students move away from their role as simply listeners and careful implementers of the instructions given by teachers. They engage in presenting ideas and take on the role of leaders at certain stages of the learning process. All these activities enable them to activate all their mental potential for the benefit of learning, through fun and play.

The comfort that students feel while learning through play increases their motivation and independence to develop an understanding of new mathematical concepts or to consolidate previously learned concepts.

According to Smith (Smith, Closser, Ottmar, & Arroyo, 2020), *mathematics teachers are already trained in the problem-solving, accuracy, and rigour required in a software-making process and the National Research Council has highlighted how mathematics and computational science are interdependent: software engineers apply the mathematical form of scientific theories, while mathematicians and scientists use powerful information technologies created by software engineers.*

Every process and activity that develops a children's critical, logical and computational thinking should be encouraged and supported. After all, there is a very close connection between critical and computational thinking, a connection that is often unified by various scientific researchers.

Stephen Noonoo, in the article"Computational thinking is critical thinking and it works at any subject" (Noonoo, 2019), said: *Computational thinking, a problem-solving process often used by computer scientists, is not so different from critical thinking and can be used in any discipline.*

And since problem-solving is the focus of learning math, critical and computational thinking is the basis of learning math, because while mathematics is the pillar of education as a whole, problem-solving is the pillar of mathematical learning.

Computational thinking, including problem-solving, systemic thinking, computational modelling, and data practices according to a basic new skill needed for economic opportunity and social mobility and is used in most areas of life (Weintrop, et al., 2016).

By preparing students, who know how to think critically, logically, and computer-wise, we will prepare citizens capable of coping with the digital world that is evolving at a breakneck pace.

3.3 Computational thinking in mathematics

When talking about developing computational thinking, it does not mean thinking like a computer or to replace the mind with a computer. *Computational thinking is a model of thinking, that is important not only in computer science and mathematics but also in other disciplines of STEM and integrated STEM education broadly* (Li, Schoenfeld, & diSessa, 2020).

Computational thinking is a way of understanding problems and it is also to describing the solution in such a way, that it can be implemented by anyone, even a machine. For example algorithms for solving square equality.

The four guides on which computational-based thinking are:

1. Decomposition (Sharing a problem in smaller portions until reaching the final solution)
2. Pattern recognition
3. Abstraction
4. Algorithms

In school math, there are often problems, whose text contains hidden requirements, for the solution of which the students should think in advance of the fragmentation of the problem into the smallest parts until reaching its full solution, and that is no other than one of the connections with computational thinking. On the other hand, students learn to stripe away from the complexity of unnecessary problems, by keeping the core of the content, and by solving the same problem by treating it more simply. They also create models and different algorithms of problem-solving that keep them in their memory all the time.

Computational thinking day by day is playing an important role in the learning process. When talking about computer science in mathematics, based on a modern definition of this concept, made by the Computer Science Association and the International Society for Technology in Education (ISTE), there are 9 skills, listed below:

1. Data collection
2. Analysis of data
3. Submission of data
4. Decomposition of problems
5. Abstraction
6. Algorithms and procedures
7. Automation
8. Parallelizing
9. Simulation

The authors of the textbooks of pre-university education mathematics, who decompose the curriculum programs, are careful to combine these 9 skills, and even introduce graphical presentations through various blocks of problem-solving algorithms. This masterfully crafted scientific material is handed down to our math teachers who should combine, depending on the themes they develop, the skills set out above, to carefully develop computer and critical thinking.

However, even the most efficient methods of teaching without the intermediary of technology can never achieve the proper motivation of students in the learning process, the development of thinking entirely and that of computers in particular.

3.4 The case study investigates the effect of learning games on the development of logical, critical, and computational thinking in students of grades 5-9 in secondary school.

The experiment will prove the benefits of math technological games (Kahoot app!) in these areas:

1. To motivate students, increase their self-esteem and stimulate collaboration between themat different levels.
2. To strengthen positive attitudes towards teaching mathematics.
3. To develop logical and critical thinking, but especially **computationalthinking** by personalizing teaching.

Through this experiment the following**hypothesis** will be tested:

The advantages of math games in general and use of the Kahoot! in teaching/learning motivate students to increase their self-esteem and cooperation at different levels.

Strengthens positive attitudes towards teaching/learning mathematics.

Develops logical and critical thinking, but especially computational thinking by personalizing teaching.

Traditional tests, performed on paper, to measure students' academic performance are graded by the teacher and do not give the child immediate feedback.Often students do not know what they did wrong and unaware of their weaknesses do not know where to focus their efforts on improving.

On the other hand, the tests, provided through the tools of digital educational technology, can be used individually by the students to assess their skills and competencies, receiving immediate feedback on the level of knowledge they possess. If the answer, given by the student, is incorrect, he/she can click to see the rationale and correct the answer. The tests can be uploaded to the application's account and can be used by other students, thus affecting the quality of education and beyond.

Kahoot! is a free, game-based learning application, that providesa pleasant environment to learn – any subject, any language, any device, for all ages!

3.4.1 Methodology

The third experiment, in contrast to the first two, which were designed by me and implemented by the teachers who had participated in the training, under my supervision, was designed and implemented by me.

The experiment took place in the non-public school "Imelda Lambertini" in Elbasan.

The study population was the students of this school and the sample was the students of grades 5. – 9.

Students of these classes underwent the experiment conducted in **two phases:**

Preparatory phase entitled"Contest of Challenges in Mathematics" lasted 13 weeks, one hour with each class each week, in the period September – December 2020, where the students of these classes each week developed one hour of mathematics plus, i.e. 5 teaching hours (out of 4 compulsory teaching hours, that are in the curriculum).

The purpose of this phase was the development of **logical and critical reasoning** in solving problems and mathematical exercises, in parallel with the development of computational thinking.

In the "Festival of Challenges in Mathematics", all students of these classes participate, regardless of their level in mathematics and each of them chooses another student to challenge him. The goal isfor students to practice and challenge themselves by solving the exercises according to the scheme in Figure 47.

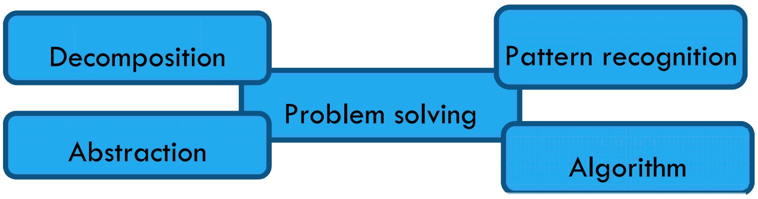


Figure 47 Problem-solving throw the computational thinking

Students are free to present the solution to the problem through posters, illustrations and even in the form of games challenging each other in their solution.

They are also instructed to organize and logically analyze the data, their relation to the requirements, the implementation of a solution plan, the creation of similarities with known solution models, the effective combination of all the steps, taken up to a certain solution point, and finalized the solution by creating the solution algorithm and interpreting the result.

Briefly, students were instructed to use computational thinking methodology to solve problems and promote cognitive thinking.

The whole philosophy of the activity developed in the first phase is based on the opinion of (BBC Bitesize, 2020) which clearly and simply explains the meaning of computational thinking: *"Computers can be used to help us solve problems. However, before a problem can be addressed, the problem itself and how it can be solved must be understood. Computational thinking allows us to do this ...; (it) allows us to take a complex problem, understand what the problem is and develop possible solutions. We can then present these solutions in a way that a computer, a person or both, can understand.*

The second phase of the experiment was studied: "The impact of learning games on the attitude of the secondary school students to the subject of mathematics and the development of their computational thinking"

3.4.2 The purpose of the second phase of the experiment

The experiment aims to show the positive results that the use of games brings in learning mathematics, not only in reinforcing positive attitudes of students' behaviours but also in the development of critical and computational thinking

The study was conducted one hour every week, during the 12 weeks of the second quarter of the school year 2020 – 2021 with students in grades 5. – 9. (150 students). The type of experiment was controlled, so the students were divided into two groups. The students of the experimental group underwent weekly quizzes, to check the mathematical knowledge, obtained in a week with the Kahoot! application.

The control group underwent the same quizzes withpencil and paper.

Kahoot!platform is constructed in a way to provide assessment in the form of a game. Questionnaires, according to the topics learnt each week, were pre-created and uploaded to an account inKahoot! The questions were constructed, based on a limited number of characters and among the alternatives, 3 were wrong and only one correct. For each question, it is set up the time limit and the manner of answering.

As these activities focus on the use of ICTs, while simultaneously activating a large number of sensors, students can solve problems with a high degree of difficulty and enable the development of critical thinking.

The data was collected based on observations of class dynamics throughout the semester as well as the results of the final test conducted on the knowledge obtained in the second quarter, compared to those of the first quarter.

3.5 Teaching/ learning process implementations

Although the second phase of the experiment will be analyzed and studied, its first phase laidthe foundations of argumentation and reasoning.

The students of both groups were monitored for the progress of the reasoning process, based on computational thinking, but the students of the experimental group were faced with thechallenge that this process would be performed not by pencil and paper, but through the software application, and against a limited time slot. (Time limit is enough to carry out all the thought processes to move towards the final solution)

The effect of time, competition, entertainment through the computer,and effects provided by the application, became part of the continuous work with the experimental group throughout the experiment. The class dynamics in this group became motivating, the students eagerly awaited the moment of the math test with Kahoot!, while the students of the other group did not show the same willingness.

Based on the topics, developed in each class, the students underwent the final test, which for the control group was conducted by pencil and paper, while for the experimental group was conducted via Kahoot!

The following criteria were taken into account for the design of the tests:

- Personal learning profiles of students in each class to personalize the process.
- Their progress from one week to the next.
- The level of difficulty of the test should be comprehensive.
- Creating exercises and problems that require the application of computational thinking
- Consider the time it takes students to complete the exercise.
- The table of conversion of points into grades for students of both groups was not used.
- The analysis of the results was done later, so that the students did not feel under the pressure of the grade while working, but focused on solving the problem.

The weekly tests as well as the final test were built on differentiated exercises, which included each of the students' levels, according to their abilities and needs. Below are the tests for each of the classes developed at the end of the period, and the results were compared with those of the first quarter.

3.5.1 Content of the written test in Kahoot!Application.

Grade 5

1. The sum of the numbers 1436+ 2364 is 200 less, than:

A. 4000 B. 3000 C. 3600 D. 4200

2. The number that contains 10 units, 5 tens and 3 hundred

A. 360 B. 153 C. 405 D. 810

3. The number 1245 is fully divisible

A. Only by 45 B. Only by 5 C. Only by 3 D. by 15

4. The sum of two numbers which are both even or both odd is:

A. Always even number B. Always odd number
C. Sometimes even number D. Sometimes odd number

5. Production of two consecutive numbers is:

A. Always even number B. Always odd number
C. Sometimes even number D. Sometimes odd number

6. Prime numbers are those numbers that:

A. Have only one divisor B. Have only two divisors 1 and themselves
C. Have more than two divisors D. Have divisor only number 1.

7. The name of a polyhedron with 12 equal edges, 6 faces and 8 vertices is:

A. Kuboid B. Coni C. Cylinder D.Cube

8. A worker in one day is paid 2400 ALL. For 30 days of work he is paid:

A. 72000 ALL B. 8000 ALL C. 7200 ALL D. 800 ALL

9. A quantity of 450 tons of wheat will be divided equally into 3 warehouses. In each of them, there will be:

A. 15 tons B. 15000 quintals
C. 300 tons less than the total quantity D. 1500 tons

10. The smallest 5-digit even number, formed by the digit 3,5,7,8,0 is:

A. 87530 B. 30578 C. 35780 D. 53078

11. The smallest 3-digit number with the same even digits that is divisible by 3 is:

A. 102 B. 201 **C. 222** D. 111

12. The value of the expression 20 +100: 20 x 4 – 3 is:

A. 21 B. 24 **C. 37** D.25

13. A car has travelled 60 km in 40 minutes. What is the speed of the car?

A.15 km/h B. 24 km/min **C. 90km/h** D. 240 km/min

14. Which of the following numbers has a quarter of the number 24?

A. 96 B. 6 C. 18 D.72

15. A right triangle with two equal sides has acute angles:

A. with a measure of 60^0 **B. with a measure of 45^0**

C. with a measure of 30^0 D. with a measure 50^0

16. The sum of the interior angles of a quadrilateral is:

A. Equal to the measure of the extended angle B. 3 times larger than the extended angle

C. 3 times larger than the right angle **D. As four times the right angle**

17. A seller bought 30 eggs for 20 ALL each, and sold them for 30 ALL each. The seller won:

A. 600 leke B. 900 leke **C. 300 leke** D. 30 leke

18. Which of the fractions is located between the quarter and the half?

A. 3/4 B. 5/6 **C. 7/20** D. 7/5

19.1/5 of a cake is eaten first, then half of it. It is left:

A.A quarter of cake **B. More than half of the cake**

C. Less than a quarter of cake D. More than a quarter, but less than half of cake.

20. A fisherman caught 3 fish, weighing 850 gr, 1.2 kg and 0.73 kg. He consumed the biggest fish. Whatis the mass of the fish that are left?

A. 158 gr B. 1.58 g C. 893 g **D. 1.58 kg**

Grade 6

1.What is the smallest even number that can be written with the digit 2,5,0,7,4?

A.20 574 B. 20 754 C. 40 572 D. 42 570

2.If we add a zero to the number 25 between the tens digit and the units digit, the number we will get is:

A.by 280 larger B.by 205 larger C.by 270 larger **D.by 180 larger**

3. The largest number, that can be placed instead of the points in the inequality:

350 – …..> 120 is:

A. 227 B. 228 **C. 229** D. 230

4. The value of the expression: 32 : (45 : 9 x 2 - 2 x 2 x 2) is:

A.12 B. 8 **C. 16** D. 4

5. The square has a surface equal to 36 cm^2.Its perimeter is:

A. 24 cm B. 6 cm C. 12 cm D. 18 cm

6. Here are 4 fractions: $\frac{3}{5}$; $\frac{5}{7}$. What is the dfference between the largest fraction and the smallest one?

A. $\frac{8}{21}$ B. $\frac{13}{20}$ C. $\frac{11}{12}$ D. $\frac{4}{35}$

7.What is the perimeter of a rectangle measuring 800 cm by 40 cm?

A. 2400 dm B. 1680 dm **C.240 dm** D. 320 cm

8. Find the sum of half of the number 640 with the quarter of 44.

A. 651 **B.331** C. 420 D. 320

9.A driver after makinga road, has still 36 km left. Find out how far the driver had travelled.

A. 18 km B. 90 km **C. 54 km** D. 36 km

10. The angle at the vertex of an isosceles triangle is 80^0. What is the measure of the angle at the base of the triangle?

A.80^0 **B. 50^0** C. 40^0 D. 55^0

11. The two classes together have 64 students. One of the classes has 8 fewer students. Find how many students each class has.

A. 34, 30 **B. 28, 36** C. 38, 26 D. 27, 37

12. 8 workers build a palace in 60 days. In how many days will 12 workers build this palace?

A. 50 B. 30 **C. 40** D. 80

13. The value of the expression $5 + \frac{1}{2} + \frac{3}{4}$,as a decimal number, can be written:

A. 5,75 B. 6,025 **C. 6,25** D. 57,5

14. There are 25 students in a class. The ratio of boys to girls is like 3: 2. The number of girls is:

A. 20 **B. 10** C.15 D. 18

15. The price of one item was 220 All. Became a price reduction of 15%. The new price is:

A. 255 leke **B. 187 leke** C. 205 leke D. 200 leke

16. If we divide a number by 7, the quotient is 4 and the remainder is 5. What is the number?

A. 23 **B. 33** C. 39 D. 27

17. The arithmetic mean of the first 5 two-digit numbers is:

A. 12 B. 13 C. 39 D.27

18. The value of *x*, which makes the equation true is:

$213 \cdot 7 + 213 = 213 \cdot x$

A.6 B. 7 **C. 8** D. 9

19. The value of the expression (2,245 + 13,755) x 100 + 1,04 is:

A. 15010,4 **B. 1601,04** C. 16010,4 D. 1501.04

20. A container filled with water weighs 5.2 kg.Filled up to $\frac{3}{4}$ of container, weighs 4.2 kg. Find the mass of the container and the mass of water.

A. container mass =1 kg, water mass =4,2 kg **B. container mass = 1 kg, water** mass = 4,2 kg

C. container mass= 0,6kg,water mass =4,6 kg D. container mass=0,8 kg,water mass= 4,4 kg

Class 7

1. LCM and G.C.D. of numbers 12, 15, and 21 are the numbers:

A. (3, 420) **B. (420, 3)** C. (6,60) D. (84,3)

2. The value of the expression: $(-3)^2 + \sqrt{25} : (-5) - (-12):(-6)$ is:

A. 12 B. 9 **C. 6** D. 14

3. With mathematical symbols the expression: "Four times the sum of a number with 7" is written:

A. **4(x+7)** B. 7 (x + 4) C. 4x+ 7 D. 4x − 7

4. Simplified form of expression: 5(2x-3y) − 2(2x + 5y) is:

A. $6x - 20y$ B. $6x - 10y$ **C. $6x - 25y$** D. $8x - 25y$

5. The acute angle of a rhombus is 70°. The measure of its obtuse angle is :

A. 80° B. 70° C. 60° **D. 110°**

6. The value of which of the expressions is $\frac{1}{12}$ smaller than 1?

A. $3\frac{1}{2}-\frac{1}{5}$ **B.** $1\frac{1}{4}-\frac{1}{3}$ C. $25x\frac{7}{15}$ B. $\frac{5}{9}of\,63$

7. The valeu of excpression: (3,012 + 0,24 x 3) : 2 x 100 : 1000 is:

A. 0,1866 B. 0,01866 C. 1, 866 D. 18,66

8. Arithmetic mean of grades 10, 9, 8, 8 6, 9, 7, 9, 10, 8 is:

A. 8.9 B. 9.4 C. 8.6 **D. 8,4**

9. A 2400-litre tank is filled with buckets that hold 1500 ml litres each. Find how many buckets are needed.

A. 2000 B. 1800 C. 1500 **D. 1600**

10. The root of the equation $\frac{6x-3}{5}=3$ is:

A. 3 B. 4 C. 5 D. 6

11. The sides of a rectangle are x and $2x$ + 1. The perimeter is 32 cm. His biggest side is:

A. 10 cm **B. 11 cm** C. 12 cm D. 13 cm

12. In a parallelogram, the measure of one of the angles is three times the mass of the other. The angles of the parallelogram are:

A.40°, 120° B. 36°, 108° C. 50°, 150° **D. 45°, 135°**

13. The decimal number found between fractions $\frac{2}{5}\ \frac{2}{3}$ is:

A. 0,36 **B. 0,52** C. 0,69 D. 0,8

14. The volume of the cube, which has a base surface of 25 cm^2 is:

A. 225 cm^3 **B.** $\frac{1}{8}$ **litre** C. 100 cm^3 D. 200cm^3

15. The two missing numbers in the sequence of numbers 4,……, ……, 25, are:

A.11,15 **B. 11,18** C. 7, 11 D. 8, 15

16. The surface of the rectangle is 48 cm^2. The height is as much as $\frac{1}{3}$ the base. The perimeter of the rectangle is:

A. 32 cm B. 40 cm C. 64 cm D. 16 cm

17.20% less than 5% of 4000 ALL is:

A.**160 lekλ** B. 40 lekλ C. 2000 lekλ D.400 lekλ

18. The equation of the line passing through the point (0; - 6) and is parallel to the *x*-axis is:

A.***y* = -6** B. $x = -6$ C. $y = 0$ D.$x = 0$

19. In one box there are pencils of two colours. Red pencils are 70%. Yellow pencils are 8 less. What is the number of red pencils in the box?

A. 20 B. 12 **C. 14** D. 6

20. What is the probability that we choose a month of the year that has 30 days, from a box where labels with all the months of the year are inserted:

A.1/3 B. 4/5 C. 7/12 D.1/2

Gifted students

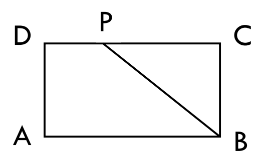
1. 4 painters can paint a fence for 20 days. 4 days after they started working, 12 painters were added to the workers' team. In how many days will the painting of the whole fence be finished?
2.

2. Knowing that $5a = 5b - 10$. Find the value of the expressions:

a) $a - b$

b) $2a - 2b - 5^2$

3.The bisector of angle *B* of the rectangle *ABCD* intersects the *DC* at point *P*, such that *DP* = 2 cm. The perimeter of the rectangle is 20 cm. Find the surface of the rectangle.



4.A driver after he had done halfway and then 25% of the remaining part of the road, still had 45 km left. Find how many km was the length of the road.

5.How many digits does the number, equal to 2^{101} x 5^{99} have?

Class 8

1. The value expression $(-0{,}1 - 0{,}9)^{101} - 100 \times (0{,}84 + 0{,}16)^{100}$ is:

A. 101 **B. -101** C. -99 D. We can't find it.

2. The prime factors in which the number 450 is decomposed are:

A.{2,3,5} B. {3,5} C. {2,5,7} D.{3,5,11}

3. If the angles of a triangle relate as 2: 2: 5, then the triangle is:

A. Right isosceles **B.obtuse isoscele**C.Acute isoscele D. None of the three

4. The simplest form of the expression: $2ab - (2a - 3ab) - 8a$ is:

A. $ab - 10a$ **B.** $5ab - 10a$ C. $-ab - 10a$ D. $5ab + 10a$

5.Which of the following numbers has a quarter of its half, equal to 24?

A. 48 B. 96 C. 6 **D. 192**

6. The value of the expression $2 - \left[\sqrt{2-(3-5)} : \frac{2}{3}\right]^2$ is:

A. -7 B. 7 C.9 D.-9

7. A car has travelled 240,000 m in 180 minutes. What is the speed of the car in km/h?

A. 60 km/h B. 80 km/h C.100 km/h D. 70km/h

8. The ends of the segment have the coordinates A (12; -3) and B (-14; 10) The coordinatesof the middle point of the segment are:

A. (2;3,5) **B. (-1;3,5)** C. (2;-1,5) D. (3;5)

9.The arithmetic mean of four consecutive even numbers is 21. The two numbers in the middle are:

A. 20, 22 B. 18, 20 C. 22,24 D. 24,26

10. The 10^{th} term of the series, presented by the general formula $a_n = 2n^2 - 50$ is:

A. 150 B. 100 C. -30 D. 250

11. The volume in cm^3 of a 10-millilitre syringe is:

A. 10 B. 100 C. 0,01 D. 0,1

12. A container of water is filled up $\frac{1}{3}$. If we add another 2.5 litres the container will be half full. The volume of the container is:

A. 5 liters **B. 15 liters** C. 12 liters D. 7,5 liters

13. The probability that a composite number will fall while rolling dice is:

A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{3}{5}$

14. The cube with a volume of 125 cm^3 has a total surface of:

A.150 cm^2 B. 100 cm^2 C.200 cm^2 D. 350 cm^2

15. Only the side x cm of a rectangle (with sides x cm and y cm) was extended by 2 cm. How many cm^2 larger is the area of the new rectangle?

A. 2 cm^2 **B. $2y$ cm^2** C. $2xy$ cm^2 D.xy cm^2

16. The slope of the line with equation $2y = 4x - 6$ is:

A. -6 **B. 2** C. -3 D. 4

17. The root of the equation $(2x - 1)(x+4) - 2x(x - 5) = 4$ is:

A.$\frac{7}{3}$ B. 12 C. -3 **D.$\frac{8}{17}$**

18. What is the height of a parallelogram that has a surface of60 cm^2 and a length of the base of 12 cm?

A. 10 cm B. 8cm **C. 5 cm** D.6 cm

19. The surface of the circumference of a circle with a perimeter of 16π is:

A. 128π **B. 64π** C. 256π D. 16π

20. Two angles have a sum of measures 120^0, and one of them is equal to 20% of the other. The measures of the angles are:

A. 100°, 20° B.96°, 24° C. 15°, 105° D. 80°,40°

Gifted students

If we add another 20% of the money with which we could buy 1 kg of first-quality rice, we can buy 3 kg of third-quality rice. What percentage of the price of first-quality is the price of third-quality rice?

Class 9

1. The intersection of two sets: $A = \{12, 24, 36, 48\}$ and $B = \{x \in N / x$ is factor of $72\}$ is:

A. {1,2,3,4,6,8,9,12,18,24,36,72} **B. {12, 24, 36}**

C. {24, 36, 48} D. {12, 36}

2. The arc of the circle stretching from the central angle of 60° has a length of 2π cm. The area of the circumference of the circle is:

A. 36π cm^2 B. 18π cm^2 C. 12π cm^2 D. 24π cm^2

3. Which of the following is NOT true for production:8 1 × 32 4?

A. 2^{18} **B. 16^4** C. 4^9 D. 8^6

4. The rhombus with diagonals 12 cm and 16 cm has a perimeter:

A. 20 cm B. 30 cm **C. 40 cm** D. 80 cm

5. Knowing that $x = 3$ is the root of equation $12kx - 72 = 0$. Find the value of k.

A. 4 B. 5 C. 6 **D. 2**

6. Complete factorization of the expression $3a^2 - 6ab$ is:

A. $3a(a - 2b)$ B. $a(3a - 6ab)$ C. $3(a^2 - 2ab)$ D. $a^2(3 - 6b)$

7. The largest natural number, that is a solution of the inequality $5x - 14 \leq 4$ is:

A. 3 B. 4 C. 2 D. 18

8. The side of a square grew 15% or 0,6 cm. Find how many cm was the length of the side.

A. 8 cm **B. 4 cm** C. 12cm D. 6 cm

9. The value of the expression $\left(\frac{2}{3}\right)^{-2} x\sqrt[3]{-64}$ is:

A. -4 B. 4 **C. -9** D. 12

10. An activity started at 15:35 and ended after 2 hours and a quarter. What time was it when finished?

A. 18:00 **B. 17:50** C. 17:40 D. 18:15

11. The volume of the cuboid based on the rectangle of dimensions 1,2 dm, 8 cm, and 10 cm is:

A. 960 ml B. 96 dm^3 **C. 9,6dm^3** D. 960 dm^3

12. The arithmetic mean of the first six divisors of the number 36, excluding the number 1, is:

A. 5 **B. 6** C. 8 D. 7,5

13. The line with equation $y = 5x - 3$ intersects the line with equation $x = 2$ in the point with coordinates:

A. (1;5) **B. (2;7)** C. (7;2) D. (5;3)

14. Simplified form of this fraction $\frac{9x^8y^6}{3x^7y^5}$ is:

A. 3xy B. $27x^{15}y$ C. $3x^{14}y^{12}$ D. $3x^{56}y^{30}$

15. A car moves at a speed of 15 m/s. How many km have this car travelled in 2 hours?

A. 72 km B. 150 km **C. 108 km** D. 200km

16. A-line with equation $y = -3x$intersects:

A. both coordinate axes B. only the x-axis C. only the y-axis D. none

17. Rotation at an angle of 180°, is also called:

A. parallel transposition **B. central symmetry**. axial symmetry D. enlargement

18. The value of expression $2a^2 + b$, where$a = 3$ and $= -5$ is:

A. 23 B. 7 C. 13 D. 20

19. The value of production $\frac{3}{5} x \frac{4}{9}$, in simplest form, is:

A. $\frac{12}{45}$ B. $\frac{14}{45}$ C. $\frac{4}{15}$ D. $\frac{27}{20}$

20. The three vertices of a square have coordinates: $A(1; 3)$, $B(1; 1)$, and $C(3; 1)$. The coordinates of its fourth vertex are:

A. (2; 2) B. (−1; 3) C. (3; −1) D. (3; 3)

3.6 Results

Observation shows that during the final test the students were focused and their reaction time to the correctness of the exercise increased.

Figure 48 Photos from experiment

As these activities focus on the use of technology and activate simultaneously a large number of sensors, students can solve problems with a high degree of difficulty and enable critical thinking development. It was interesting that high school students (grades 8,9) who underwent testing with Kahoot!, prepared by the teacher for a set of topics, taken over a week, with 45 minutes each week, in the subject of mathematics, were inspired to build their Kahoot! with knowledge of different subject matter and fields by utilizing the library (Encyclopedias) and extending the game with other classmates, who were not selected to undergo this test.

9th-grade students went even further by introducing other elements to make this game more interesting and interactive. It was noted that the students felt motivated to collect data, conduct research to build structured questions, to generalize cases.

On the other hand, students who were subjected to traditional testing, and from the beginning were sceptical of becoming part of Kahoot!, grew interested in how this platform works and in the meantime prepared questions with alternatives to getting ready for Kahoot! on their own. The selected students to be tested with Kahoot were of different levels.

The motivation and results were fantastic. Students who did not have math as their preference significantly increased their interest in this field. Students who, although they do not show evaluated results in mathematics, built their own Kahoot!, although with simple exercises.

As it is shown on the graph (Figure 49), the average age of students studied was 13,04 years. The mean is 13 years.

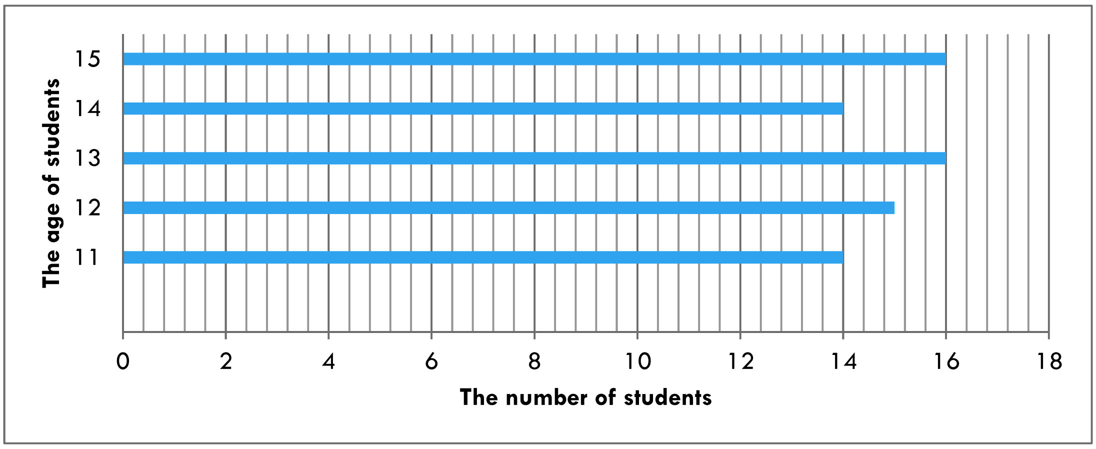


Figure 49 The age of the students

The chart below (Figure 50) shows the result of the group that was tested with Kahoot!

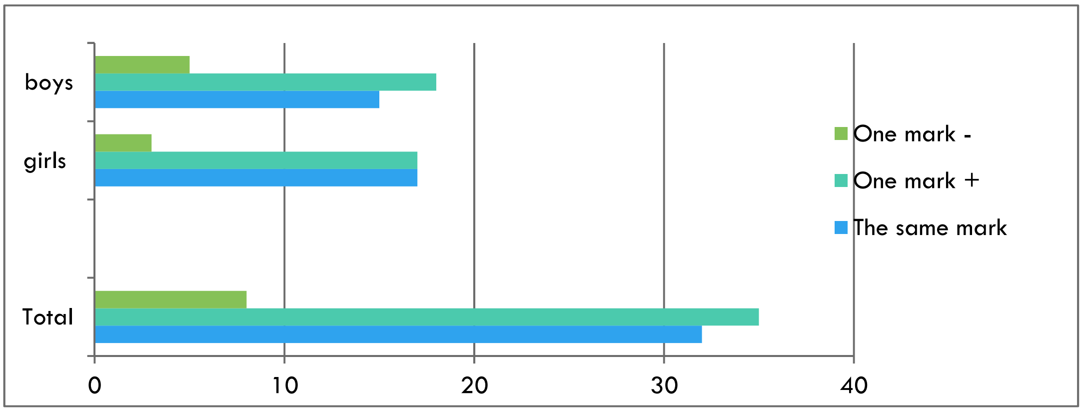


Figure 50The results in experimental group

3.6.1 The results of the experimental group

About 47% of the participants increased the grade received, compared to the second quarter in the continuous evaluation, on average by 1mark.

43% of them maintained the same mark, while 10% experienced a slight decrease (by one mark). No significant differences were observed in the gender of the students surveyed.

The chart below shows the result of the group that was tested with pencil and paper. (Figure 51)

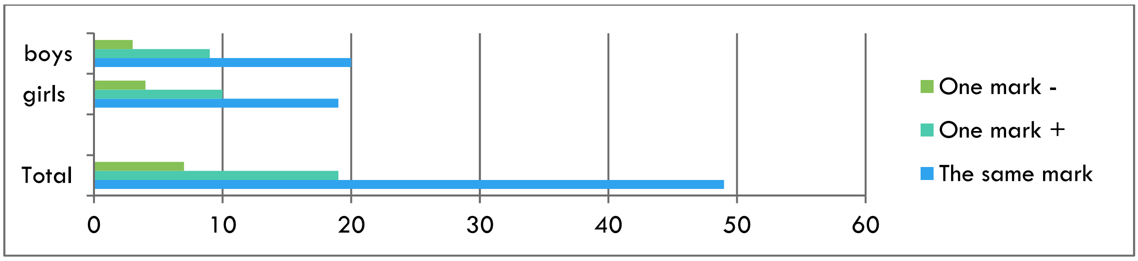


Figure 51The result of the group that was tested with pencil and paper

3.6.2 The results ofthe control group

25% of the students increased their grade obtainedby 1 mark, compared to the second quarter in the continuous evaluation, about 66% of them maintained the same grade, while 9% suffered a slight drop (one markdown). No significant differences were observed in the gender of the students surveyed.

The following chart shows a summary of the results of both groups of students in the test (Figure 52).

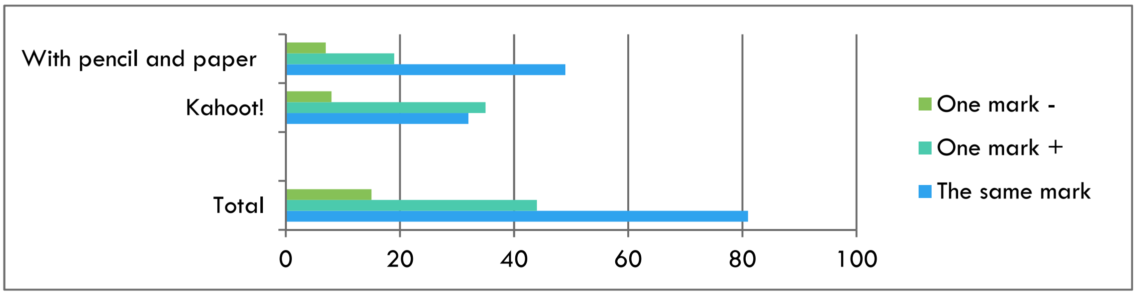


Figure 52 Summary of the results of both groups of students in the test

The graphical presentation shows that the number of students who have increased their score by 1 mark, is greater in the group tested with Kahoot!, than that of students tested with pencil and paper.

This difference has resulted in a higher number of students who have maintained the same mark compared to the previous quarter in the group of students tested according to the traditional method.

What was noted during the study was that the implementation of this game for a certain period with a particular group of students brought about an increase in all components that pertain to class dynamics (Figure 53).

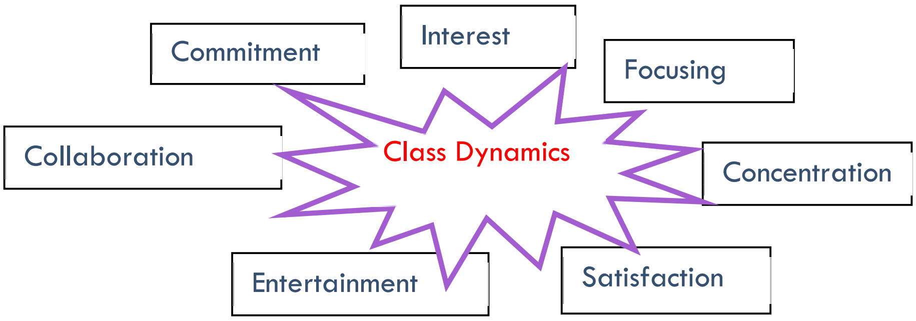


Figure 53 Class dynamics

Students who were subjected to Kahoot! were more vital in class discussions during other math classes, asked questions, and discussed topics they would have in the coming week, and how the questions might have been formulated.

The interesting finding of this study was especially the moment of Kahoot! 's creation by themselves.

It was impressive that during the drafting of the questionnaires on the Kahoot platform! students gave the right time to those problems that required the decomposition of the initial problem. This makes it the reason that they are in the right direction to judge and solve even more difficult problems and it shows that computational thinking is developing in the right direction.

In some particular students, laconic ways of formulating the questions without sacrificing the essence of the question were noted, thus achieving a high level of computational thinking.

An aspect to be evaluated in Kahoot!, is also the competitive one, which enables students to target the classification head by increasing the classroom dynamics and motivating them to do the job before reaching the goal.

In this way, they are encouraged to think critically.

3.7 Conclusions and recomandations

1. The time of implementing the experiment served as a time of reflection and reinforcement of knowledge, as the students were pushed to build their Kahoot!, and therefore had to systematize the acquired knowledge.
2. The cooperation between them deepened, as in the group they repeated and deepened the mathematical concepts by creating questions about the scientific material received in class.
3. The limited number of characters that could be used to create a question on Kahoot! streamlined their thinking without compromising scientific accuracy.
4. The fact that each question required a solution time and a correct answer, forced them to solve all the exercises written in their Kahoot in advance and then serve them to their friends for the solution.
5. The experiment served to disconnect students from the traditional way of learning, re-dimensioning their concept of using a cell phone or laptop not only as a tool to talk with

friends but as an effective tool through which you can progress in a subject, that for the most of students it is seen as the "forbidden apple".

The study was conducted in a non-public school where the material base and computer laboratory are modern, but the state of public schools is far from this reality.

For technology to be a good supportfor computational and critical thinking development, all actors must cooperate, starting from the teacher, who must support teaching methods with modern educational technology, and school administrators, who must provide the necessary technological base for the learned, but above all ambitious state policies, to bring all these actors together.

Until this cooperation is ensured, it takes a lot of work and a lot of time, since the pace of educational technology is not parallel to the base of materials available to public schools for experiments of this type.

On the other hand, today's students may not be in SmartBoard classrooms and modern labs in public schools, but 90% of them have a smartphone or tablet. If they are guided by competent, creative teachers who aim to personalize the learning process at all times, even while playing, they *can bring the change the world expects from this generation*. Why not, even while having fun, *play is the engine of true learning and the highest form of research*. (Albert Einstein)

It should be a priority for our policymakers to design policies to develop the critical and computational thinking of our students and to provide educational technology tools as a very good opportunity to use, not only the material served in the text but also other ways of learning, such as online learning, e-learning, virtual libraries, virtual universities, e-learning, e-journals, etc.

4 CONCLUSIONS

The personalized lesson with the students of the three experimental groups had different characteristics, which in summary are:

In the first experiment, the treatment of additional teaching materials, in addition to that used as the first source material (students' book and worksheets), accompanied by the competencies of the field developed by each task or group of tasks, as well as with differentiated content created the opportunity for students to work independently.

Using SmartBoard even after learning, in groups or pairs, speaks to the flexibility that students need to be created to learn at any time. Their work in groups according to defined activities increases interaction and empowers them from the point of view of decision-making. The plan of activities implemented in the experiment is a model of how to integrate the problem-solving steps by personalizing the teaching and, at the same time,to develop all the competencies in the field.

The second experiment provides a model of how to start a personalized learning with identifying the strengths and weaknesses encountered in the pre-test. The structure of the experiment, the detailed plans according to the students 'levels, the exercises and problems of applied character, the solution of which was supported by GeoGebra, bring positive changes in the class dynamics as well as in the students' interest to deal with mathematics, in addition to increasing performance of students. The shift of math learning, beyond the classroom walls, to their home computers, being practised with the GeoGebra app speaks to a boost of formal from informal learning in their homes.

The third experiment brings another model of how the game-based mathematics learning could be personalized by simultaneously developing critical and logical thinking and computational thinking. In this experiment, it was taken into account that students can learn anywhere, and in any way. The game in Kahoot! served not only to highlight each week students' weaknesses through simple weekly quizzes but stimulated their desire to create own Kahoot! with answers, and massified the game process by dealing with math.This activity encouraged students to raise their level of reasoning.

The findings of the study show that to personalize mathematics learners, the teacher needs:

- Knowledge of students' learning profile
- Creating detailed plans and strategies according to the level of students
- Finding effective teaching methods and techniques
- Support students with source material differentiated in content
- Choosing the right ICTs tools to support the whole process.

The study can serve as a reference point for the process of personalized learning as a whole, or the development of teacher training modules, with topics related to personalized learning.

Also, it can serve as a new practice to personalize learning math through play, even on their mobile phones or from the computers they have at home.

CHAPTER 6. INFLUENCE OF ICT-SUPPORTED PERSONALIZED LEARNING OF MATHEMATICS ON THE MOTIVATION TO LEARN MATHEMATICS

Technology in the classroom is NOT the end goal. Enabling learning everywhere is the goal

Andrew Barras

Motivating students to learn is a very delicate process. Perhaps the most delicate process of the mathematical education process. He can be considered without a doubt the most decisive factor in the success of students in mathematics. Transformation of teaching hours into stimulating and collaborative environments, supported by technologically efficient educational tools, working with clear and structured goals, and supporting each student according to his abilities and needs, motivates students to learn mathematics and consequently increases the performance of the tire. In this chapter, we will see how all these factors have influenced the increase of motivation of students who underwent the experiment.

1 INTRODUCTION

The self-esteem of the members of a society is an important condition for a peaceful society inclined towards progress. The goal of the educational system is to educate motivated generations, who set positive and ambitious goals for themselves and want to move forward. This is also one of the goals of mathematics in the school system.

Motivation is the process that initiates, directs, and maintains goal-oriented behaviour. In short, motivation gets you to act in a way that brings you closer to your goals. (Cherry, 2022)

Teaching in general and teaching mathematics, in particular, is a very complex process. It should bring together the teaching profession, the dedicated professional, the charismatic leader, the penetrating psychologist in the most delicate parts of their behaviours, the inspirer, and the distributor of positive energy to motivate students in the learning process. In addition to all these qualities, he must be a creative artist to amplify the whole process with the right technological educational tools.

To study motivation in the content domain of school mathematics need to examine the relationship that exists between mathematics as a socially constructed field and students' desire to achieve. (Middleton, 1999)

The field of mathematics is very tangible and applicable in everyday life, so care must be taken to refer to this connection to reality whenever possible.

However, how much do our math teachers manage to motivate their students and how much does technology influence to support this process of motivation and self-assessment?! Let us analyze the results of the questionnaire, taken by students,who underwent the experiments.

2 METHODOLOGY

After experiments in the three schools, a questionnaire was submitted to the students of the experimental groups. It investigates the frequency of using ICT tools to develop different activities related to the teaching of mathematics in their schools. It also collected students' opinion regarding the importance of the use of ICT in the teaching mathematics, to make it as comprehensible and entertaining as possible. In addition to these components, the questionnaire also aimed to collect data on how the use of ICT in teaching mathematics increases cooperation, creativity and all other components that affect students' motivation. Another goal of the questionnaire was to collect the opinion of the students about the characteristics that a lesson should have, in order the students to participates there with pleasure.

The method used was that of the survey, while **the instrument** used was a questionnaire with mixed questions, closed and open. In terms of closed-ended questions, the Likert scale was chosen to measure statements of agreement, frequency, and importance. Each of the five responses has a numerical value which is used to measure the attitude under investigation.https://www.simplypsychology.org/likert-scale.html.

The open-ended questions aimed to gather opinions about the purpose why each of them taught mathematics as well as the components that should be completed in a mathematics lesson, in which they would be willing to participate.

So as we said, the **target population** was the students who underwent the experiment, i.e. 216 students from the three schools where the experiments were applied.

3 RESULTS, ANALYSIS AND DISCUSSION

The students included in the study belonged to 3 secondary schools which participated in the experiment.

Their average age was 13.2 years.Of these, 48% were girls and 52% were boys.The students of these schools had the following technological tools:

- 36% of them had only a smartphone;
- 8% had only tablets;
- 12% had both a computer and a tablet;
- 24% had smartphones and computers;
- 12% had smartphones, and tablets and even used their parents' computers;
- 2% owned them all;
- 6% had access only to their parents' computers and had neither a smartphone nor a personal tablet.

The students all had access to the Internet, both on their smartphones and on the Internet lines dedicated to their families.

Today's students spend a lot of time with their technological tools, for a variety of reasons.The results of the question "How much time do you spend with your technological equipment per day?" are presented inFigure 54.

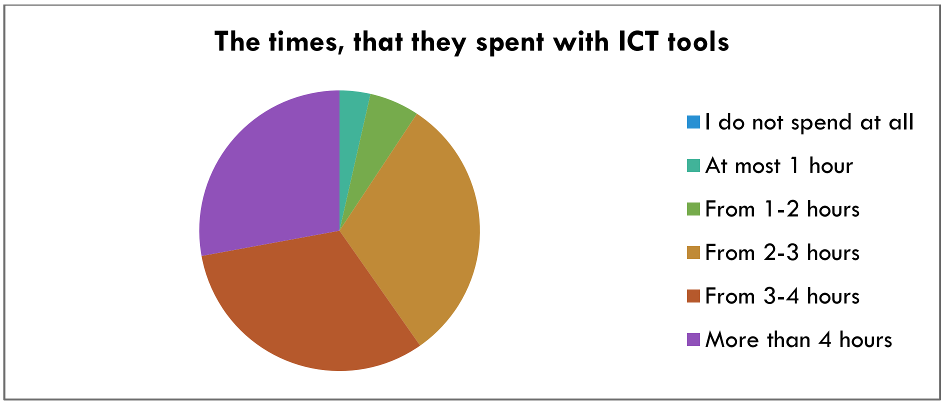


Figure 54 The time that they spend with technological equipment per day

3.1 Use of ICTs tools owned by students to learn mathematics

The time that students spend with technology tools is huge, but how often do they use these tools to perform activities related to math learners?

The results in Table 15shed light on the frequency of activities they perform to teach mathematics.

Table 15 The activity they do with ICTs tools

The activity they do with ICT tools	Never (1)	Rarely (2)	Sometimes (3)	Often (4)	Always (5)	Mean
To follow tutorials that explain mathematical concepts or exercises.	15	66	51	53	31	3.1
To play mathematical games.	57	79	32	23	25	2.4
To copy math assignments sent to me via WhatsApp, or other communication networks.	74	72	55	15	0	2.1
To install applications that help me to solve homework.	70	37	72	18	19	2.4
To follow math tricks or to expand my knowledge of the world of math.	39	63	69	34	11	2.6
To play fun games with a mathematical character.	45	104	22	27	8	2.2
To make presentations of mathematical projects that we are obliged to do.	2	25	71	19	108	4.1
To send homework by email to the math teacher or to upload them to the virtual platform we work with.	3	60	50	36	67	3.5

The activity they do with ICT tools	Never (1)	Rarely (2)	Sometimes (3)	Often (4)	Always (5)	Mean
To be discussed in mathematical forums.	90	35	51	12	18	2.1
To follow Facebook or Instagram or other pages or App-s, that have to do with math.	90	45	59	17	5	2.1
To explain or listen to the solution of the exercises to my friends on a videocall.	82	35	38	34	27	2.5

Results show that the activity, that students perform mostly with their technological tools,are following the tutorials, where mathematical concepts are explained, or performing exercises, similar to their tasks.Using digital technologies for these activities is a very natural process considering that the students often solve many issues, that have to do in their daily life, through ICT tools.

Also, a considerable number of students responded that the technology is used for sending homework by email to their teacher or for uploading it to a virtual platform with which they work.

This practice has its origins in the time of the Covid-19 pandemic when students conducted online learning even in many schools in Albania, it continues to be preserved, often applying blended learning.

An equally large proportion of them has shown that they use technology to make presentations on mathematical projects that they are obliged to do.

3.2 ICTs tools in math learning to increase students' self-esteem and motivation

How does the use of technology affects the elements that increase motivation in learning mathematics?

The results, reflecting students' opinions, are shown in theTable 16.

Table 16. The impact of ICT tools on some of the elements that motivate students to learn mathematics.

Using ICT tools in teaching math	Not at all (1)	Not really (2)	Undecided (3)	Somewhat (4)	Very much(5)	Mean
Increases the desire to learn mathematics	4	21	18	95	78 4.0	
Makes math an easier subject to understand	5	24	15	86	85	4,0
Makes math more fun	2	18	13	98	85	4.1

Using ICT tools in teaching math	Not at all (1)	Not really (2)	Undecided (3)	Somewhat (4)	Very much(5)	Mean
Strengthens cooperation between students	8	22	25	75	86	4.0
Increases creativity and helps the imagination	2	17	21	85	91	4.1
Increases students' self-confidence and self-esteem	7	19	24	71	95	4.1
Illustrates the connection that mathematics has with the world around us	3	12	23	103	65	3.9

The collected answers lead to the conclusion that students see technology as a potential motivating tool, because, for them, the use of ICTs tools in explaining mathematics increases the desire to deal with mathematics, makes it simpler and more understandable, makes it fun learning, enhances collaboration, enhances their creativity and support their imagination, strengthens their self-confidence and self-esteem, illustrates the connection between mathematics and the reality.

Some of the answers, that most of the students provided to the question "Why do you learn math?" are:

- Helps my progress in all other subjects;
- Develops my work discipline;
- Helps me to solve various problems in the market;
- Helps me to create diverse mosaics and applications;
- Helps me to understand stoichiometric calculations in chemistry;
- Helps me to understand some biological processes that have essentially mathematical models;
- Helps me to understand the axis of time in history;
- Helps me to understand the calculation of the musical notes;
- Helps me to understand the calculation of calories I consume and gain;
- I will need it when I continue my studies in architecture;
- To help parents in their businesses;
- To become a mathematics teacher;
- Develops my logic, and helps me to solve problems related to my daily life;
- Helps me to reason critically.

Their answers show that the students are essentially motivated to learn math.So, if the teachers treat this internal motivationthrough the use of digital tools, in combination with external motivation factors, the students will succeed in mathematics.

Students were asked to describe the characteristics of a math lesson they would love to attend. They list components such as:

- Relating the mathematical concept to real-life situations;
- No time limit of 45 minutes per lesson;
- Solving problem situations in different ways;
- Lesson filled with activities where the protagonists are the students themselves;
- Returning attention to difficult topics;
- Organizing group work;
- Research-based lessons;
- Differentiation of the level of tasks;
- Continuous parallel repetition, etc.

In addition to the above components, in almost all the answers some elements connected the teaching of mathematics with the use of ICT tools. Some of the answers were:

- Record the teacher's explanation and publish the link with her explanation in the virtual classroom, for a second listening.
- Use of SmartBoard to illustrate the explanation of lessons as well as to present group work.
- Short quizzes, in different applications at the end of the lesson.
- Introduction of fun elements such as curiosities on various topics or even from the life of mathematicians displayed on the SmartBoard.
- Creating online quizzes by the teacher and entering the virtual classroom, to be practised in the afternoon.
- Use the SmartBoard, especially in geometry-themed classes, to provide clear images, and enhance the visualization of 3D figures.
- Information on additional links provided by the teacher is to be consulted in the afternoon, to support the understanding of the concepts taken in a lesson.
- Use the GeoGebra application as often as possible to support math classes, especially those related to function.
- Encouragement to create math games like Kahoot!, to have fun learning.

The answers reflect the degree of satisfaction with this experience after listing the components of a lesson, and those elements that were present during the development of the experiment.

ICT tools, used wisely in teaching mathematics, increase students' motivation and confidence that mathematics is a subject that serves to understand and explore reality and the deeper it is studied, the more beautiful it becomes.

These answers were in sync with the results of the answers regarding the experience of the use of ICTs in the math lessons, rated from "Very boring (1)" to "Very interesting" (5) (Figure 22).

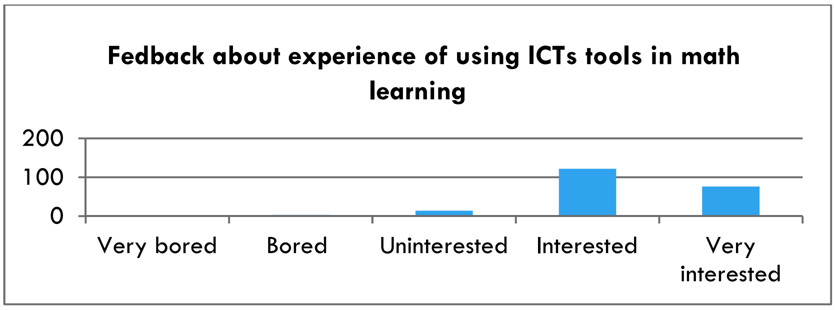


Figure 55 Students, feedback about the experience of using ICTs tools in math

This feedback received on a subject such as mathematics, which in the opinion of most students is considered a difficult subject, clearly shows that if our students are treated with intelligence and care in all aspects of their development, the results are very good. On the other hand, increasing self-esteem and motivation to learn mathematics confirms the fourth hypothesis of the thesis that "The use of ICT in teaching mathematics, to personalize learning, brings higher results in students 'mathematical thinking and at the same time increases students' self-esteem and motivation "

4 CONCLUSIONS AND RECOMMENDS

The positive results and confirmation of the hypotheses, raised in the three experiments, were not accidental.

The positive results, obtained during the experiment, are a product of a well-thought-out combination of learning activities developed with the help of ICT tools and perfect planning of teachers' work. These two components were the two strong points for achieving these results.

Some of the activities that influenced the achievement of these positive results, both in increasing their intellectual performance in a personalized way and the positive attitudes towards the learning process as a whole, were:

1. Pre-creation of a personal profile of students' learning by delving into the characteristics, abilities, strengths and weaknesses of each of them.
2. Continuous reference in PPL, as well as good knowledge of students' learning modalities based on their strengths and weaknesses, to personalize learning while encouraging students' desire to learn mathematics according to their potential.

The positive energy of the students during the mathematics lessons, as well as their commitment to learning, was in sync with the commitment of the teachers and their enthusiasm.

Increasing students' motivation to engage in mathematics means educating students that:

- To have a clear purpose for which they learn mathematics and for this purpose, they also know the benefits that come from it.
- To believe in their abilities.
- To be creative and make continuous efforts to develop their creativity.
- To be engaged in every moment of the learning process and beyond.
- To investigate by working together until a mathematical result or conclusion is drawn.
- To challenge themselves and their friends.

- To be curious to find and interpret different solutions to situations from reality.
- To have developed self-esteemand.

Bringing students to this state is not done by chance, as their motivation to learn mathematics is the base, without whichthe development of their mathematical competencies would be impossible.

To be successful in the process of motivation, the math teacher should offer to the students:

1. The figure of the professional passionate in the learning process.
2. Lessons that are creative and always bring new situations away from the routine, with plans and activities that students have clear. They should be such that they are always amazed at the creativity that supports them as well as the instruments used to achieve the goal.
3. Active teaching methodology so that they feel involved in each stage of learning and do so willingly.
4. Positive social situation enabling collaborative learning techniques.
5. A friendly classroom environment where they want to be welcomed creates the conditions for them to feel valued and involved.
6. Clear panoramas on the connection that the mathematical concept has with the real world.
7. ICT tools that ambitiously support all of the above.

Conclusions, Discussions and Further work

ICT tools are strongly influencing education in general, but in particular – mathematics education. Technological education tools are playing an important role in modernizing mathematics learning by shifting learning, from teacher-centred learning to student-centred learning, as well as competency-based learning. Shifting the focus to active teaching/learning methodologies would be impossible without ICT tools.

1 Benefits that ICTs tools offer supporting the teaching/learning of mathematics

The benefits that ICT tools offer in the field of teaching mathematics are:

1. Facilitate and accelerate mathematical operations by expanding the space to be oriented and focused more towards the application of logical structures for solving problems related to the application of mathematics in real life.
2. Reinforce concepts through multimedia by playing an important role in sensory components and allowing the use of a wide variety of learning modalities.
3. Develop abstract thinking, strongly supporting problem-solving strategies, project-based learning, question-based learning, etc.
4. Provide computer simulations as illustrations for solving problems that present a high degree of difficulty.
5. Visualize the learning process by increasing the level of understanding and facilitating the transition from concrete to abstract thinking.
6. The variety of programs, software and computer platforms it offers enables students to learn at their own pace.
7. Increase students' motivation to deal with mathematics.
8. Develop computational and analytical thinking, as skills and competencies acquired through computer programs are the basis of computational thinking in all areas in general and mathematics in particular.

2 Regarding the current context in Albania.

It is a fact that state policies have been and continue to be very ambitious regarding the implementation of ICT in teaching. This is reflected both in the curriculum frameworks of the mathematics curricula and in the National Strategies for the Development of Education in general.

These ambitious policies are focused on creating the appropriate infrastructure that starts from setting standards for ICT equipment in schools, investing in digital equipment and their technical maintenance, to developing school-level policies.

This whole process requires ensuring the implementation of the strategy in all schools in Albania, transparently monitoring the whole process, as well as preparing teachers to use the infrastructure in a scientifically justified way.

However, the current situation in Albania needs urgent intervention in this direction.

This intervention should be done in three main directions in parallel:

1. Teachers are continuously trained in two directions – firstly to personalize teaching, based on active methodologies, and, secondly, for meaningful use of ICT in schools, not only as a

facilitator of their daily work at school but as a supporter of the learning process as a whole, supporting students to advance according to their abilities and needs in the learning process.
2. Creating a European model of teachers with the characteristics set out in "The UNESCO ICT Competency Framework for Teachers" by setting up a structured system of policies and priorities, based on an analysis of the strengths and weaknesses of our teachers in terms of competence digital.
3. Taking measures to acquire digital competence in terms of teaching mathematics at the university, coordinating work with universities that prepare teachers to introduce as a special curriculum the preparation of technological didactic tools for teaching.

Although measures have been taken one after the other and successive reforms have been made to bring the model of the Albanian teacher closer to the model defined by UNESCO, there is still much to be done in this direction.

The PISA results from year to year speak about a modest change that mathematics education has undergone in Albania.

Two of the fundamental changes that have been made in the curriculum of mathematics education in Albania, competence-based learning and learning in the centre require in the first place the strengthening of all the potentials that our students possess, starting from the emotional ones to the intellectual ones in the field of mathematics. For this empowerment to happen, there is a need to design policies that promote personalized learning by training teachers to personalize mathematics learning as well as the use of ICT tools to support the whole process.

The challenge of learning mathematics in the 21st century around the world is personalized learning, therefore it remains a challenge for the education of mathematics in Albania.

3 PERSONALIZED LEARNING IN MATHEMATICS

Although personalized learning is defined, explained and analyzed in different ways, there are some common characteristics:

- Personalized learning is competency-based learning.
- It put the student at the centre of the learning process.
- It optimizes the pace of learning and learning approach based on students' abilities and needs.
- It orients to move forward with confidence and courage to achieve the final goal.
- It supports and promotes logical, critical and abstract reasoning.
- It encourages creativity and collaboration.
- It develops multidimensional skills that shape the contemporary citizen with fair and rational judgments about the world around him.

The process of personalizing learning is a process that requires continuous and quality commitment.

The mathematics teacher, while personalizing the lesson, simultaneously performs multidimensional activities, such as the teaching process itself.

Some of them are summarized in Table 17:

Table 17. Range of tasks to be completed by the math teacher as he/she personalizes the teaching and learning of mathematics

Teacher	Important tasks to be completed by the math teacher to personalize learning
Recognizes	• The needs and interests of students. • Their strengths and weaknesses. • Their multidimensional skills (starting from personal ones to those in the field of technology). • How long does it take for everyone to master each of the mathematical concepts? • Gaps they have concerning previously learned knowledge. • Learning modalities as well as their learning profile.
Selects	• Appropriate active teaching and learning strategies. • Technological platforms and materials will be implemented during active teaching. • Topics that will be intertwined, while conceiving learning on a project basis or in group work. • The model they will design to implement in teaching. • Techniques and tools with which to intervene in students' weaknesses to improve them, while working hard to strengthen their strengths. • Combinations of personal skills and those in the field of technology will combine to realize the process of teaching mathematics.
Prevents	• The situation of despair in the face of difficult concepts. • The situation of pessimism is associated with the thought "I am not made for learning mathematics". • Abandoning the desire to do the math. • Getting away from the core while you are learning math. • Deepening the weak points in teaching mathematics. • Superficial thinking while doing math.
Guides	• At the right time. • In the right place. • At the right level. • According to the need and abilities of each (to personalize mathematics learners). • The path to critical, logical, and analytical thinking. • The platform or technological tool to be used.

Teacher	Important tasks to be completed by the math teacher to personalize learning
Encourages	• Independent thinking to structure meaningful tasks. • Creative thinking further develops social skills as well as what they already know from experience. • Logical thinking, to organize their mathematical thinking while trying to successfully realize the product of their work. • Building in-depth knowledge of mathematical content, thus developing critical thinking. • The spirit of cooperation and the desire to work in a group. • The desire to learn math.
Creates	• Competent citizens in the field of mathematics and beyond. • Collaborating citizens, who are creative and know how to collaborate to build together skills and competencies. • Global citizens who know and value their values. • Citizens that are capable of decision-making from, a managerial point of view. • Citizens who know how to work in a team, manage time, synthesize information, and use technological tools. • Students and managers of the mathematics learning process are being led and mentored by him.

Personalized learning in mathematics through ICTs strongly influences simultaneously in two directions:

- in active learning;
- as a catalyst for the cognitive activation of students.

ICT tools in teaching/learning mathematics are an added value of the learning process in the digital age.

The application of ICT tools in mathematics teaching/learning is multifunctional. In addition to motivating all levels of students, it has many other functions in many aspects of teaching. Let's mention some of them:

1. Improves students' understanding of mathematical concepts.
2. Supports teaching through visualization.
3. Favors coollaborative learning.
4. Strengthens students' self-esteem.
5. Assists gifted and talented students.

Personalized learning in mathematics supported by ICT tools, is a process that should be placed as the focus of educational policies for the training of mathematics teachers in Albania, to perform this mission so complex.

ISTE has set very high standards for students, ranging from "Learning to use technology" to "Transformative learning with technology", but achieving these standards requires qualified teachers and ICT tools to support the process.

Some aspects that would take on considattion in the current situation in Albania are:

- The types of ICT tools that teachers have at their disposal for teaching in the schools where they teach.
- Personal perception of the use of ICTs tools in mathematics learning.
- Their professional motivation is to be trained in the use of ICT in teaching.
- Personal experiences with using ICTs tools to personalize learning.
- Difficulties they encounter when using ICTs tools in teaching mathematics.

4 THE IMPORTANCE OF IMPLEMENTING ICTS TOOLS IN TEACHING AS SOON AS POSSIBLE. THE SITUATION IN ALBANIA.

Digital technologiesprovide a powerful tool in the mathematics learning process, as well as a supporting "hand" to empower cognitive processes of personalized learning.

The support that these ICTs tools give to personalized learning in mathematics enables each student to develop his intellectual potential in his own unique way.Reform to move away from traditional ways of teaching/learning once and for all must be fundamental.

Based on competence learning, where the learner should be at the centre, teachers should embrace active, technology-supported learning methods, moving from behavioural theories to active learning theories. On the other hand, they need to apply curricula and active teaching methods that enable students not only to develop mathematical competencies but also to use them. In this way, they will once and for all get away from teaching that is based on unconstructive content.

It is important that the use of ICT tools in teaching mathematics with student focus and the development of critical and logical thinking, are not seen as "saving" tools that lead to a constructivist approach, or an orientation towards learning based on competence, but as a support in terms of encouraging creativity, critical reasoning and logical reasoning.

Lesson time is not only the time, or physical space available to the math teacher, but it is also the moment when the formation of mathematical concepts comes to life, it is the time for the formation of students' mathematical culture for life.From this point of view, this environment should be complemented and treated carefully not only from the point of view of educational technical support but also from the point of view of planning.

On the other hand, the planning of each lesson should be done carefully, it should be constructed and planned in such a way that the learning outcomes for each student are optimal.In this planning, care must be taken to find the right moment, where the intervention through ICT tools brings to each of the students, depending on the learning modalities and his learning profile, the maximum benefit.

The care should be taken that this use is not seen as a **selfish** use, but as a **meaningful** use.

The results of the questionnaire shed light on the training needs of mathematics teachers in Albania as well as the access they have to ICTs tools in the schools where they work. Analysis of the answers, given concerning the questionnaire, shows that the variety of ICT tools, available to mathematics teachers in the schools, is at a minimal level. The teachers have their own technological devices in use in everyday life – mobile phones, laptops or personal computers, but only through them,they could not properly carry out an entire process such as active teaching through technology.

Half of teachers use technology rarely or not at all in teaching and this is an indicator that should be taken into consideration by educational policy makers, to create policies that: firstly provide training for teachers in using ICT effectively and, secondly, provide schools with a good technological base to be used in teaching.

This process should not be seen as a sporadic process in specific schools or certain periods, but as a whole ongoing process in the form of institutional policies organized with detailed programs. In addition, the framework of these policies should require also training of the mathematics teachers regarding the digital skills and competencies that they should possess, according to their needs, at the licensed training agencies that exist in Albania.

5 TEACHER TRAINING – OBLIGATION TO PERFECT ONESELF, TO PERFECT TEACHING.

All human beings are subject to a continuous learning process. Life itself, in its entirety, is a learning process. During this learning process, the innate ability to learn is improved by expanding both the range of ways of learning, as well as the skills and competencies acquired through this process. Students come into life with an amazing potential for learning, a potential that must be pursued, nurtured and expanded with extraordinary care.

They possess the gifts and talents that *sleep* within them, and they wait for someone to carefully *awaken* them, to take care of them to grow and to be gradually perfected. Teachers have this delicate and multidimensional mission.

The future of society as a whole depends on the way teachers are formed, on their coherence with all the changes that occur in their professional aspect as well as on the technological developments that enable their professional growth.

They must undergo continuous training, to be updated with both technological developments in the field of education, as well as innovations in teaching while staying in coherence with global developments in all areas of society.

Although teachers need to know their students' learning styles to adapt them to their teaching strategies, incorporating as many techniques and methods as possible into mathematics teaching can enhance their intellectual abilities, although sometimes the strategy used in teaching may not suit the learning style of any of the students. The variety of techniques and methods used in teaching mathematics, educates every student, gradually a variety of learning styles, although they may not be as powerful as the main style of learning.

The training delivered to the mathematics teachers showed that, despite the work experience of mathematics teachers, despite their high academic level, they have to constantly apply teaching methods that combine psycho-pedagogical and methodological aspects with the support of ICTs tools. Student success in mathematics will only be achieved if the teachers carefully combine the following four components:

- Personalized learning
- Adapting teaching strategies to students' learning styles.
- ICT support
- Optimal management of all stages of the learning process

In Albania, which is a developing country, mathematics can play an important role in transforming the whole society. For this reason, continuous training of teachers regarding mathematical content, pedagogy and active teaching methodologies should be a priority of educational policymakers.

6 RECOMMENDATIONS REGARDING TEACHERS' PIECES OF TRAINING IN THEFIELD OF DIGITAL COMPETENCIES

The experience, taken during the pilot teachers' training, leads to some recommendations about their design and delivery:

1. Possession of digital competencies for the application of educational technologies in the teaching process should be an obligation for teachers and to be controlled and followed with transparency by the authorities, which in the qualifying tests that take place after (5, 10, and 20 years of work) to include questions that test teachers' digital skills.
2. Monitoring of Teacher Training Agencies, for a more equal distribution of the developed training, giving the training related to ICT tools in education the appropriate space or reorganising of the training agencies by profiling them to offer training in 1 - 2 categories.
3. Monitoring of the training they perform after the training plan has been approved by the competent bodies, bringing under control a fair distribution of training according to the fields.
4. Establishment of teacher mentoring structures by qualified teachers towards the use of ICTs tools in teaching, motivating the latter for the service provided.

7 CONCLUSIONS ABOUT THE EXPERIMENTS

7.1 SmartBoard experiment

Differentiating student levels through comprehensible schemes or using colour graphics conveyed to students a clear, understandable, visually appealing learning material, increasing curiosity as well as the duration of their concentration.

Introducing the material interactively, in addition to students' performance on the results, also influenced their emotional behaviours and the approach they began to take to mathematics after the experiment.

The use of SmartBoard increased the interest in learning mathematics, as well as the collaboration between the students. The math teacher indicated that she often found them in small groups by reviewing solution schemes or other learning materials stored on the SmartBoard.

Using SmartBoard increased the teacher's availability of time in classroom-planned activities, as ready-made materials prepared at home could be downloaded directly to the desktop from her email and presented on the classroom SmartBoard.

In short, in addition to increasing student performance, the teacher's performance is also increased.

The basic principles of personalized learning were rigorously followed in both groups, paying attention to the development of the key competencies, which were specific to the topics developed during this quarter.

Also, the treatment of concepts and the construction of new knowledge, based on a problem and further the treatment of problem-solving, according to the defined steps, made it possible for students to progress at their own pace by simply analyzing, for students at the basic level, down to the most specific analysis for senior students.They gave thoughtful opinions and the quality of the judgment increased from week to week.

With increasing, judgment came motivation and an increased desire to deal with mathematics.

Mathematics in Albanian pre-university education is presented as a field of dual character.

1. On the one hand, through counting, measuring, modelling and geometric concepts, students discover the world around them and provide the language and basic techniques for managing many aspects, including those of everyday life.
2. On the other hand, with the power of abstraction, logical argumentation and the beauty of proof, it is presented as an intellectual discipline and as a source of aesthetic pleasure.

The strengthening of both of these characteristics is done in the first place by following the personal learning profile of the students, to design the technique of strengthening methods as well as having the support of ICTs tools.

Its dual character requires the development of subject competencies, which were realized in the experiment by differentiating the levels of students according to abilities and needs, differentiating the content and finding a way to reach each level of students in a motivating and attractive way.

7.2 GeoGebra's experiment

The main conclusions, extracted through the GeoGebra experiment are:

1. The use of GeoGebra software brought its effect first on the student's results in the post-test. The post-test results in the experimental group increased by 0.84 based on the arithmetic mean, compared to the pre-test, while in the control group the average of a slight increase level by 0.08 compared to the pre-test.
2. The motivation and enthusiasm of the students of the experimental group during the teaching of mathematics increased more.
3. The use of the program, in student stories, was extended in the afternoon at home on their laptops or tablets. Thus, GeoGebra expanded the boundaries of learning, moving from formal to non-formal education in the classroom in their homes and leisure discussions.
4. Increase student employment, not only within the classroom but also between classes by separating from each other and explaining in different ways what they learn from GeoGebra.
5. The students also benefited a lot in the field of ICT, greatly facilitating the teaching of the mathematics teacher, to personalize the teaching.
6. Using software shortened the time you would waste with graphic and sketch constructions, giving students the advantage of engaging in understanding and modelling as key subject competencies.
7. Using GeoGebra software was an experience that professionally enhanced even the teacher, who during all this time was practised in advance at home to bring out the best she could for the students.

Although this software helped to visualize the linear function, it is very important to maintain a balance between lessons, where educational technology tools can be used, and those where students had to practice manually.On the other hand, the use of GeoGebra as a technological teaching tool to support personalized learning can be used to extend to all classes and topics, both in geometry and geometric transformations and in other topics.

The practice of using GeoGebres in teaching requires training for students on how the software works and the areas of its use. It is recommended that in the subjects of informatics, in parallel with the subject curriculum, there are special topics, with instructions, for the way of using didactic software for learning.

This would increase students' autonomy to learn, as well as enable them to work with technological educational applications and software even in their spare time at home. In an ideal situation, students should spend with their computers, phones or tablets at the same time, as the lessons taken, with the computer to make it part of non-formal math education.

7.3 Kahoot! appexperiment

The conclusions from the Kahoot! The experiment could be summarised as follow:

1. The time of implementation of the experiment served as a time of reflection and reinforcement of knowledge, as the students were pushed to build their Kahoot!, and consequently had to systematize the acquired knowledge.
2. The collaboration between students deepened, as in groups they repeated and deepened the mathematical concepts by creating questions about the scientific material obtained in the classroom.
3. The limited number of characters with which they can be used to create a question in Kahoot!, rationalized their thinking, without compromising scientific accuracy.
4. The fact that for each question the solution time and the correct answer had to be predicted, forced the students to solve in advance all the exercises they had written in the Kahoot! created by themselves, and then to test their friends. This *knowledge transfer chain* not only strengthened their sense of responsibility but also increased cooperation between them.
5. The experiment served to disconnect the students from the traditional way of learning, re-dimensioning their concept of using mobile phones or laptops not only as tools to talk with friends but as effective tools through which to progress in a subject, which, for most of them, is seen as the forbidden apple.

The study was conducted in a non-public school where the material base and computer laboratory are modern, but the situation in most public schools is far from this reality.

For technology to be good support for computational and critical thinking, it is necessary to bring all actors together. Concrete educational policies are needed to support schools with educational technology assets, trained teachers are needed to apply educational technology in teaching, as well as a passion to be innovative in everything that serves effective teaching.

Until this cooperation is ensured, it takes a lot of work and a lot of time, since the pace of development of educational technology is not moving in parallel with the material infrastructure available to public schools for experiments of this type.

8 THE MOTIVATION OF STUDENTS WHO UNDERWENT EXPERIMENTS

The positive results and confirmation of the hypotheses, raised in the three experiments, were not accidental.

The positive results obtained in the experiment, are the product of a well-thought-out combination of learning activities, developed with the support of ICT tools, and perfect planning of teachers' work. These two components were the two strong points for achieving these results.

Some of the activities that influenced the achievement of these positive results, both in increasing the intellectual performance of students in a personalized way and in positive attitudes towards the learning process as a whole were:

1. Pre-creation of a personal profile of students' learning by delving into the characteristics, abilities, strengths and weaknesses of each of them.
2. Continuous reference in PPL, as well as good knowledge of students' learning modalities based on their strengths and weaknesses, to personalize learning while encouraging students' desire to learn mathematics according to their potential.

The positive energy of the students during the teaching of mathematics, as well as their commitment to learning, was in sync with the commitment of the teachers and their enthusiasm.

Increasing students' motivation to engage in mathematics means educating students that:

- Have a clear purpose for which they learn mathematics and for this purpose, they also recognize the benefits that come from it.
- Believe in their abilities.
- Are creative and make constant efforts to develop their creativity.
- Are engaged in every moment of the learning process and beyond.
- Are collaborators, to research until a mathematical result or conclusion is drawn?
- Challenge themself and their friends.
- Are curious to find and interpret different solutions to situations from reality.
- Have developed self-esteem.

However, the development of these conditions is not done by chance, as the motivation of students to learn mathematics is the first, without it the development of mathematical competencies is impossible.

To be able to succeed in this process, the math teacher should offer to the students:

1. The figure of the professional passionate in the learning process.
2. Lessons that are creative and always bring new situations away from the routine, with plans and activities that students have clear. They should be such that they are always amazed at the creativity that supports them as well as the instruments used to achieve the goal.
3. Active teaching methodology so that they feel involved in each stage of learning and do so willingly.
4. Positive social situation enabling collaborative learning techniques.
5. A friendly classroom environment where they want to be welcomed creates the conditions for them to feel valued and involved.
6. Clear panoramas on the connection that the mathematical concept has with the real world.
7. ICT tools that ambitiously support all of the above.

9 DISCUSSIONS AND FURTHER WORK

1. **Summary of the key findings and interpretations**

The results indicate that:

The existing technological base in Albanian schools and the digital competences of Albanian teachers are not at the right level to personalize the teaching / learning of mathematics supported by ICT.

The continuous training of teachers in terms of learning modalities and the support of ICT tools, for contemporary active teaching in mathematics, is very important.

The results obtained from the experiments developed in 2 non-public schools and in a public school, fulfill the expectations and support the hypotheses raised in this study.

There is a positive correlation between the personalization of mathematics learning through ICT, with students' performance, motivation and self-confidence to deal with mathematics.

A positive result that was observed in the study was the commitment of the teachers who developed the experiment to develop their digital competences, even in a self-taught manner. At the end of the experiment, they possessed knowledge and digital competences regarding the software they used, which went beyond the knowledge they had received in training, or that they needed to guide students to use the software.

2. The implications of the study

The results of the study, not only support the existing theories on personalized learning, but they go further by bringing a model of a process that requires a deep understanding of the skills, needs and talents of the students, to be combined with many other psycho pedagogical factors, on one hand and the continuously updated implementation of educational technology innovations for personalization of teaching and learning math on another.

The results shed light on the supporting role that educational technology has in teaching, emphasizing the fact that an efficient process of personalized learning in mathematics requires that everything be studied and well-planned in detail.

This well-planned study begins with a deep knowledge of the students' skills, needs and talents, and not only with their psycho-social characteristics, it continues with detailed and well-studied plans according to their level, as well as the choice of supportive educational technology.

The study measures the fact that the process of personalized learning requires, first of all, trained teachers who know how to choose among the variety of technological educational tools, the one that is most productive to support the personalized learning of mathematics.

The data that the study brings contribute to a clearer understanding of the dominant role of the competent teacher, not only in the scientific content of the subject, but also in the digital one, to choose the supportive educational technology that is most productive to support personalized mathematics learning.

3. Limitations of the study

The generalizability of the results obtained in relation to the technological base that exists in Albanian schools is limited since the questionnaire in 18 large schools was also completed by 2 teachers of the same school. This limitation could not be eliminated, as the questionnaire simultaneously collected data not only on the technological base but also on several other issues such as:

- To investigate the readiness of mathematics teachers to use ICTs in the classroom
- To investigate the interest they have in being trained in the use of ICT in teaching mathematics.
- To investigate the personal experience with the use of ICT in the classroom.
- To identify the difficulties encountered by mathematics teachers in Albania in using ICT in teaching mathematics.

But even if we consider this limitation, the number of teachers who completed the questionnaire remains a large number and does not affect the reliability of the results in relation to all the investigations carried out.

4. Recommendations for practical implementation or further research

Although in he teachers' questionnaires, there participated teachers from all over the country, it is still difficult to organize a country-large experiments with students in public schools.

As the private schools has better ICT infrastructure and equipment, it is easier smoothly to transfer to student-centerd personalized teaching of math by the support of ICTs.

Educational system in Albania is highly centralized. To transfer the results from the pilot experiments in the three volunteree schools, it is nessecary 1) to provide a large, nationally supported teachers trainings; 2) to ensure appropriate infrastructure and conditions in schools for moderlization of educational process, and 3) to organize nationally directed experiment in a representative sample of the Albanian schools.

The authors researchers plans include offering similar teachers training courses to the Ministry of Education, with the support of the Elbassan University.

The first pilot results of the study could be used to initiate larger pedagogical experiments in chosen schools in coordination and mediation of the Ministry of Education, aiming ot support the Strategy for switching from content-based to the competency-based education.

AUTHOR'S CONTRIBUTIONS

SCIENTIFIC AND APPLIED CONTRIBUTIONS

Scientific contributions

Personalized learning is a field that dates back to the 1800s and has undergone changes over time. Personalized learning in mathematics must precede personalized learning as a whole. Personalized learning in mathematics is a modern educational approach, based on the strengths, skills and competencies of students, on the needs and weaknesses of students, supported by the opportunities, techniques, methods, and ICT tools, offered to students for their progress, with self-esteem and motivation. It is seen as a very subtle process that goes beyond differentiated learning, which is based on the needs of students, and beyond individualization, which is based on their pace.

Through this PhD thesis, personalized teaching in mathematics is addressed from a modern perspective, based on the support that ICT offers for personalizing mathematics teaching.

The scientific contributions that come from this dissertation are:

5. A detailed analysis has been made of the parallel evolutions that mathematicians and ICT thinkers have known, as well as the mutual influence they have had on each other.
6. The compatibility of the infrastructure with the regulatory framework for the competencies of teachers in Albania has been investigated, and it has been analyzed how close they are to the ICT competence framework of UNESCO for teachers.
7. An analysis has been made on the evolution and challenges of ICT in Albania as well as the policies of the Albanian government to implement ICT in the field of education.
8. The approaches of the regulatory framework to the mathematics curriculum based on competencies in Albania were analyzed, as well as the modest changes that came from its implementation, in the results of Albanian students in PISA.
9. A systematic review of personalized teaching in mathematics was done. They are the main teaching methods for the personalized tutoring course in mathematics.
10. The four components of personalized learning have been extensively discussed, explaining their characteristics with examples.It is analyzed the role that personalized learning plays in the mathematical form of students in lower secondary education.
11. A national questionnaire was designed and developed to investigate the attitude of Albanian mathematics teachers towards the support of ICT tools, to personalize the teaching and learning of mathematics, focusing on:
    - Personal perception of the use of ICT in teaching/learning mathematics.
    - Teachers' interest in being trained in the use of ICT in teaching/learning to support personalized learning in mathematics.
    - Personal experiences with the use of ICT in teaching.
    - Difficulties encountered by teachers in their daily life, to use ICT to personalize the teaching of mathematics.
12. Semi-structured interviews were conducted with mathematics teachers.The interview aimed to investigate if the difficulties of mathematics teachers at work are related to:
    - the mathematics curriculum

- with the quality of textbooks
- with students' interest in mathematics
- with the difficulties, they have to adapt to the rapid ICT developments
- with the lack of training.

13. A detailed analysis of learning styles, accompanied by illustrations and examples from the mathematical context is followed by a series of advice that mathematics teachers should follow to personalize mathematics learning.
14. The entire process of personalization of learning mathematics is seen as a complex process related to teaching strategies, learning styles, and application of digital technology in learning mathematics and is illustrated with the "Pyramid" of personalized learning" (Figure 33) which expresses its connection with other components.
15. A methodology has been developed for training the teachers for the personalization of mathematical education with the support of ICT. It is equipped with relevant assessment teaching materials. The study provides:
    - A practical model of using a SmartBoard in personalized mathematics teaching.
    - Amodel of using the GeoGebra software for handling the concept of function in the eighth grade.
    - A model of using the application Kahoot!, to develop computer and critical thinking supported by mathematical problems that develop logic and rational thinking.

All the models brought for the developed experiments are supported by detailed plans according to the student's level as well as authentic mathematical problems for the identification of gifted and talented students.

16. A questionnaire was designed and developed for the students who were subjected to the experiment, which investigated the frequency of using ICT tools to develop different activities related to the teaching of mathematics in their schools. The questionnaire collected data on how the support of ICT tools in teaching mathematics increases cooperation, creativity and all other components that affect motivation. Another aspect that was intended to be investigated was the characteristics that a lesson should have, which students would be happy to attend.

Scientific-Applied contributions

Some of the models or templates which are scientifically justified and which can be especially applied in the field of mathematical education can be mentioned as follows:

1. Recommendations for educational policies in Albania to intervene in the direction of the implementation of ICT in education, where the three main directions where this intervention should be made are defined.
2. A list of important tasks that must be completed by the mathematics teacher to personalize the lesson has been made. The list contains tasks that the teacher fulfils and performs these activities: recognizes, chooses, prevents, directs, encourages, and creates.
3. A questionnaire was designed to investigate the learning styles of mathematics for students.
4. The questionnaire designed to investigate the access to technology that mathematics teachers have in Albania can be used to investigate the situation at a later time.
5. The results of the analysis of this investigation and the relevant recommendations are to be used in research of the same nature.

6. Training resources on learning styles and the role technology plays in supporting them in personalized learning.
7. Recommendations for education policymakers regarding teacher training to improve digital competencies
8. Detailed thematic plan on the concept of functions in the eighth grade, together with exercise models and the key mathematical competencies they develop.
9. Tests (pre-test and post-test) were specially designed to investigate the performance of students before and after the experiment, together with the evaluation table.
10. Logical questions for testing students from grades 5-9, in the application Kahoot!
11. Lists of authentic problems according to different levels of students are offered to teachers as a guide to guide new teachers to select different problem situations depending on the key competencies that each problem situation develops.
12. Questionnaire to measure the degree of motivation of the students, who were subjected to the experiments, as well as the results of the analysis of the collected data.

Mathematics teachers in Albania are very interested in training in the field of active teaching as well as in the support that educational technology offers to personalize teaching. Likewise, the professional networks of mathematics teachers conduct continuous training concerning active teaching methodology.

The teachers who take part in the training can train the teachers in their schools thus increasing the number of trained teachers. On the other hand, this model could serve teacher training agencies in Albania to provide personalized training for students in mathematics, and not only, supported by educational technology.

LIST OF AUTHOR'S PUBLICATIONS, RELATED TO THE TOPIC OF THE PHD THESIS

D. Starja, Personalized Teaching of Mathematics through ICT and the Personal Perception of Teachers of Using ICT in the Learning Process, Proceedings of EDULEARN21 Conference, Publisher:IATED Academy, 2021, pages:5973-5979, ISSN (online):2340-1117, ISBN:978-84-09-31267-2, doi:doi.org/10.21125/edulearn.2021

D. Starja, The Impact of Learning Games on the Approach of Secondary Grade Students Towards the Subject of Mathematics and the Development of their Computational Thinking, Proceedings of EDULEARN21 Conference, Publisher: IATED Academy, 2021, ISSN (online):2340-1117, ISBN:978-84-09-31267-2, doi:doi.org/10.21125/edulearn.2021

D. Starja, N. Nikolova, **From Pencil and Paper to ICT in Mathematics Teaching. An Overview of the Role of ICT in Mathematics Teaching in ALBANIA**, EDULEARN20 Proceedings, 2020, pages:2635-2644, ISBN:978-84-09-17979-4

Diana Starja, Nikolina Nikolova, Bederiana Shyti, Personalized Learning in Math, through Problem-Solving, and the Use of ICT, Education and New Developments 2020, editor/s: Mafalda Carmo, World Institute for Advanced Research and Science (WIARS), Portugal, Publisher:inScience Press, 2020, pages:304-308, ISSN (print): 2184-044X, ISSN (online): 2184-1489, ISBN: 978-989-54815-2-1

D. Starja, B. Shyti, N. Nikolova, The Role of ICT in Improving Problem-Solving in Teaching Mathematics, EDULEARN20 Proceedings, Publisher: IATED Academy, 2020, pages:2651-2659, ISBN: 978-84-09-17979-4, Ref, PhD

D. Starja, N. Nikolova, Importance of Logical-Mathematical Algorithms In School Mathematics, EDULEARN19 Proceedings, editor/s: L. Gómez Chova, A. López Martínez, I. Candel Torres, Publisher: IATED Academy, 2019, pages:4482-4489, ISSN (print):2340-1117, ISSN (online):2340-1117, ISBN:978-84-09-12031-4, doi:10.21125/edulearn.2019.1124

DECLARATION OF ORIGINALITY OF THE RESULTS

I declare that the present dissertation contains original results obtained in my research with the support and assistance of my scientific advisers. The results obtained, described and/or published by other scholars are duly and in detail cited in the bibliography.

This dissertation is not applied to obtaining a scientific degree in another higher school, university or scientific institute.

Signature:

(Diana Starja)

APPENDIXES

APPENDIX 1: QUESTIONNAIRE ABOUT ICT'S ROLE IN PERSONALIZED LEARNING IN MATHEMATICS.

This questionnaire is about the use of ICT (Information and Communication Technologies)by math teachers for personalized teaching.

Personalized teaching is one of the challenges of the 21st century in education, consequently, it is also a challenge for teaching mathematics.

The questionnaire aims to collect data regarding the frequency, quality and variety ofuse of ICT to personalize the teaching of mathematics, which in generalmeans the design, development and implementation of teaching strategies and techniques taking into account the abilities and intelligence of each student.

The questionnaire aims to identify:

a) The types of ICT tools you have available for teaching in your schools.
b) Personal perception of the use of ICT in mathematics learning.
c) Teachers' interest in being trained in the use of ICT in teaching.
d) Personal experiences with the use of ICT in the classroom.
e) Difficulties encountered by math teachers in using ICT in teaching mathematics.

Please read and answer each question carefully.

All responses are anonymous and will be treated with strict confidence.

Your contribution is very important to this study.

THANK YOU for your cooperation and precious time!

I. Socio-demographic information

1. Age (years):

a) 25 - 30 years
b) from 31 to 35
c) from 36 to 40
d) from 41 - 45
e) from 46 to 50
f) from 51 to 55
g) from 56 to 60
h) From 61 to 65

2. Gender

a. male
b. female
c. prefer not to answer / other

3. Your school is part of the regional directorate of education:

a) Korçe
b) Durres
c) Fier
d) Lezha

4. Your school is:

a) Public school
b) Non-public school

II. Professional context

5. The average number of students per class is:

a) Less than 10
b) 10-15
c) 16-20
d) 21-25
e) More than 25

6. The number of academic hours per week in teaching math:

a) 4
b) 8
c) 12
d) 16
e) 20
f) More than 20

7. Your seniority at work as a math teacher is:

a) Less than 1 year
b) 1 - 3 years
c) 4-10 years
d) 11-20 years
e) 21-30 years
f) More than 30 years

8. The school, where you currently work is:

a) In a rural area
b) In an urban area

9.Which of these technological devices are available in your school?

	Yes	No
Computer lab		
Laptops for teachers		

USB for teachers		
CD/DVD		
Video – projector		
Whiteboard		
SmartBoard		
Printers		
Photocopies		
Interactive tables		
Tablets		
Tablets only for students		

10. How long have you been using information and communication technology (ICTs) to teach math:

a) Less than 1 year
b) 1 to 3 years
c) 3 to 6 years
d) 6 to 9 years
e) 9 to 12 years
f) More than 12 years

11. In your school, during the conducting of math classes, the access of teachers and students to ICT infrastructure is as follows:

	Never	Rarely	Sometimes	Often	Always
Small groups of students are equipped with 1 computer and the internet					
Each student is equipped with a computer and the Internet					
Computer and the Internet					
Only the teacher uses the computer and the internet, linked to interactive whiteboards for the whole class					
Both you and the students use computers and the Internet					
Neither teachers nor students use the computer and have access to the Internet					

12. Does your school have a school domain with personal accounts for teachers and students?

a) Yes

b) No

13. Can students use personal technological equipment to benefit the development of the lesson?

a) Yes
b) No
c) Only if requested in advance

14. How many times a week do you use a technology device to teach math:

a) Never
b) Rarely
c) Often
d) All the time
e) Depending on the knowledge I have to explain

15. You use ICT tools in the classroom to teach math in this direction:

	Yes	No
For the demonstration of a learning process, in the first phase		
For the construction of knowledge		
For the strengthening of knowledge		
Depending on the concepts I have come to explain		

III. Personal perception of the use of ICT in mathematics learning.

16. In your school, during the conducting of math classes, the access of teachers and students to ICT infrastructure is as follows:

	Never	Rarely	Sometimes	Often	Always
Small groups of students are equipped with 1 computer and the internet					
Each student is equipped with a computer and the Internet					
Computer and the Internet					
Only the teacher uses the computer and the internet, linked to interactive whiteboards for the whole class					
Both you and the students use computers and the Internet					
Neither teachers nor students use the computer and have access to the Internet					

17. The use of ICTs tools in teaching mathematics is mostly used for:

	Yes	No
Ongoing evaluation(formative).		
Summative assessment		
Evaluation of the subject portfolio		

18. You use ICT in teaching math: (you can choose more than one alternative)

a) To work with students with learning difficulties
b) To work with the talented students
c) To work simultaneously with all students
d) To enable everyone according to their abilities
e) To enable everyone according to their interests

19. In your opinion, the use of ICTs tools in teaching/learning math is:

	Yes	No
One of the many resources that support teaching methods to realize a personalized learning		
It's the best way of teaching a personalized math learning		
It is a necessity of the time in which we live, for the personalization of learning mathematics as part of personalization in many spheres of life		

20. You use ICT tools in the classroom to teach math in this direction:

	Yes	No
For the demonstration of a learning process, in the first phase		
For the construction of knowledge		
For the strengthening of knowledge		
Depending on the concepts I have come to explain		

21.For you, personalized math teaching is about:

a) Simply with the trinomial: student-computer-knowledge.
b) With multi-sensory learning experiences, based on the promotion of collaborative and communicative skills, according to the abilities and needs of students.

22. If the educational technology base of the school where you work would be very good, learning math for youwould be:

a) More qualitative
b) Nothing would change,
c) It would be more personalized
d) I do not have an answer

IV. Teachers' interest in ICT training in teaching.

23. The type of training for professional development in the field of ICT that you have received is in the following fields:

Using the Internet for teaching	Yes	No
Use of basic computer software like Windows, Exel, PowerPoint, etc.		
Informative training related to mathematics learning platforms and applications		
Advanced training in the use of mathematical applications and Software		
Dynamic software that enables interactive geometry, interactive algebra, interactive statistics and arithmetic		
Virtual teaching.		
Advanced courses for using the internet, creating web pages, etc ...		
Specific training for the use of devices such as Whiteboards, SmartBoard, laptops, and tablets in teaching mathematics		

24. In which of these social networks do you exchange professional experience related to the use of ICT in teaching? (You can choose more than 1 answer)

a) Facebook
b) Google +
c) Twitter
d) Linkedin
e) Whatsapp
f) eTwinning

25. The number of professional training provided by the education authorities in the field of ICT during the last 5 years is:

a) 0
b) 1
c) 2
d) 3
e) 4
f) 5
g) More than 5.

26. The nature of your professional training in the use of ICT in teaching is:

	Yes	No
Mathematics teaching training, through ICT provided by schools or educational institutions		
Self-taught training with tutorials available on the Internet		
Professional training from others, (according to my interests) regarding the opportunities offered by ICT for personalized teaching in mathematics		

27. The person conducting your ICT training at your school is:

	Yes	No
An experienced teacher in the field of ICT or didactic responsibility of the school		
The teacher who develops the subject of Informatics		
Another expert, out of school		
The school institution does not train us in the field of ICT		

V. Experience with the use of ICTs in teaching

28. How often do you do the following activities?

	Never	**Rarely**	**Sometimes**	**Often**	**Always**
Search the internet to find teaching materials					
Use applications to prepare video presentations for teaching					
Create personalized tests for each group of student levels					
Creating mini-tests with exercises and problems for homework or class					
Sending homework or evaluating a homework project sent to your email					
Test a single topic, a group of topics, or an entire chapter with a test written in google.doc					
Sending homework or evaluating a homework project sent to your email.					
Formulate questionnaires to get student feedback on a teaching method or					

	Never	Rarely	Sometimes	Often	Always
technique you have developed in the classroom that is important to you to test.					
Creating files to organize, student portfolio works.					
Search the internet for information to work differentiated with talented students.					

29. Which of the following types of materials did you use when teaching your classes?

a) Existing materials online, from edited educational resources
b) Materials that exist in the school database
c) Material that you created and then hard copy materials For example sheets with selected tasks of 3D models for demonstration brought with you flesh
d) Materials that exist in the school video library (DVD, or CD)
e) Ready-to-use links where you have entered in advance and found that there is material for personalized work.

30. Which of these Digital Learning Management Systems have you used for teaching?

a) Google Classroom
b) Akademi.al
c) Century Tech
d) EkStep
e) ClassDojo
f) Edmodo
g) Other (specify)

31. Which of this software do you know about personalized math teaching?

a) MathNation
b) CanFigureIt Geometry
c) Desmos
d) GeoGebra
e) Virtual Nerd
f) Brilliant
g) PhET Interactive Simulations
h) CueThink
i) Khan Academy
j) I do not know how to use any Software that has to do with teaching math
k) Use other software that supports math teaching (specify)

VI. Difficulties encountered by teachers in using ICT in teaching mathematics

32. Indicate to what extent the following factors negatively affect personalized learning in mathematics through ICT

	Not at all	A little	Enough	A lot
The insufficient number of computers in the school.				
Poor Internet quality in school.				
Insufficient number (or non-existence) of Whiteboards or SmartBoards.				
The poor functional condition of school computers.				
Insufficiency of competencies and skills I possess in the field of technology.				
Insufficient knowledge I have of the use of mathematical software in teaching.				
The limited number of teaching platforms related to mathematics, translated into Albanian.				
Lack of digital textbooks.				
Insufficiency of ICT lesson models to personalize mathematics learning.				
Existence of time limits within which we must develop the lesson.				
The existence of low school budgets, for the creation of digital classrooms and technologically supported environments.				
Parents' scepticism about learning math through ICT.				

32. How well do you master the activities related to the use of ICT in teaching mathematics?

	Not at all	A little	Enough	A lot
Write a test with mathematical exercises, which contain fractions, power operations, geometric figures and bodies, and graph construction.				
Use e-mail to communicate with students, or colleagues in the function of teaching.				
Make and edit digital photos, or other images, and upload them to the documents they need to create tests or project presentations.				

	Not at all	A little	Enough	A lot
Create a database on your computer for later use.				
Create a questionnaire or test in google doc format. and then interpret the data.				
Save a file to your account drive.				
Organize the material you have on your computer into folders, and subdivisions, according to the classes where you teach, or according to the level of students for different classes. etc.				
Draw a graph using a spreadsheet, after entering the data into the cells of a table.				
Create an animated video or audio presentation.				
Use effectively all the applications contained in the Googleclassroom platform.				
Create or maintain a math blog or web page, to use as a resource for personalized work with students of all levels.				
Download and install on your computer, math software, that you need to use for personalized work with students.				
Download the materials you need, from secure educational sources, and store and process them afterwards.				
Use a SmartBoard for teaching with pre-prepared material or using links that enable attractive teaching.				
Save the materials used on a SmartBoard as a database for years to come.				
Use the KAHOOT! application, or any other application for testing logical questions or any type of questionnaire.				
Teach students to have their account in the KAHOOT application !! or in any other mathematical game, to use it for mathematical reasons.				
Suggest math games to students as an effective way to personalize math learning				
Use effectively all the applications contained in the Googleclassroom platform.				

	Not at all	A little	Enough	A lot
Create or maintain a math blog or web page, to use as a resource for personalized work with students of all levels.				

APPENDIX 2: QUESTIONNAIRE FOR TEACHERS REGARDING THE EFFECTIVENESS OF THE TRAINING.

A. Personal data

1. Your gender is:

- ☐ Male
- ☐ Female
- ☐ Other

2. Your age in years is:

- ☐ 25 - 30
- ☐ 31 - 35
- ☐ 36 - 40
- ☐ 41 - 45
- ☐ 46 - 50
- ☐ 51 - 55
- ☐ 56 - 60

3. Your work experience (in years) is:

- ☐ 0 - 5
- ☐ 6 - 10
- ☐ 11 - 15
- ☐ 16 - 20
- ☐ 21 - 25
- ☐ 26 - 30
- ☐ 31 - 35
- ☐ More than 35 years of work

B. Professional context

4. Your knowledge of student learning styles and how technology supports them before training was:

- ☐ At least
- ☐ Good
- ☐ Very good

5. In your work, do you first identify the learning styles, to further adapt the teaching strategies and the technological tool that you can use

- □ Ever
- □ Rarely
- □ Not always
- □ Frequently
- □ Always

6. Do you think it is important to identify students learning styles to then build your work based on their styles?

- □ It does not matter much as you use a variety of strategies and methods in teaching and consequently, each student will find themselves
- □ It’s important
- □ It's very important
- □ Another thought….

7. Do you build a PPL in advance for the students in the classrooms where you teach to personalize the math learners?

- □ ever
- □ Rarely
- □ Not always
- □ Frequently
- □ Always

8. If you create a PPL, does it often refer to your work?

- □ Ever
- □ Rarely
- □ Not always
- □ Frequently
- □ Always

9. Can you, during your work, individualize math learners, thus teaching each student according to the style that he/she feels more comfortable with and more productive?

- □ No, because a large number of students do not allow it
- □ Yes, because the classes we teach have a small number of students.
- □ Not always
- □ Sometimes
- □ Always

10. How often do you "pluralize" your math lesson (you pass on the learning topics to the students in different ways, according to their type of learning)?

- □ Never
- □ Rarely
- □ Not always
- □ Frequently
- □ Always

11. Have you used any of the platforms discussed in the training, or others to amplify the teaching strategy in line with the student's learning style?

- □ Yes
- □ No
- □ I used others (specify platform and style)

12. Do you think your students will benefit the most from adapting to the skills and needs they have if you use technology to enable them to learn by the student's learning style?

- □ No
- □ Does not make any major difference
- □ It will make a big difference

13. Evaluate from 1 (I did not benefit at all) - 5 (I benefited a lot) the effectiveness of this training concerning learning styles.

- □ 1
- □ 2
- □ 3
- □ 4
- □ 5

14. Evaluate from 1 (I did not benefit at all) - 5 (I benefited a lot) the effectiveness of this training concerning the platforms and technological tools that were discussed.

- □ 1
- □ 2
- □ 3
- □ 4
- □ 5

15. Rate from 1 (not at all important) - 5 (very important) training related to the use of technology in mathematics teaching

- □ 1
- □ 2
- □ 3
- □ 4
- □ 5

Thank you for the answers given!

APPENDIX 3: TEST FOR IDENTIFYING STUDENTS' LEARNING STYLES.

Name ....

1) I remember more information from a lesson:
   a) If the teacher explains the math lesson in a pre-recorded video at home
   b) Listening to and watching a video taken on the Internet
   c) By reading it, take notes sketched individually
   d) If the teacher while explaining engages me to do an experiment or something similar
2) In class you are more attentive:
   a) When the teacher explains on the board and at the same time sketches concept maps
   b) When you work on my class notebook according to her instructions
   c) When you read, sketch and repeat in the book the information received.
   d) When we carry out projects and can use body or model construction
3) During the lesson:
   a) You focus on the dress of the teacher or other friends around you.
   b) You are disturbed by every kind of sound I hear around
   c) You are attentive to all the details that the teacher brings about the topic of the lesson
   d) You have more attention to the way I take notes in your notebook
4) For you, a math test is simple if:
   a) Includes graphics, illustrative figures, or diagram constructions
   b) You would be able to read the requests aloud
   c) There are exercises for solving which you have to read, extract data, and sketch.
   d) It is a test where you have to build figures with geometric tools or calculate the surfaces of figures
5) If you realize a project in mathematics, for you the favourite would be:
   a) A project that connects art with mathematics
   b) A project that connects mathematics with music
   c) A project for which you would have to search for material through books or web pages
   d) Build a mock-up of geometric figures and bodies
6) The task you like to do while we are working in a group is:
   a) Introduce group work
   b) To read the conclusions drawn by the group
   c) Keep track of all group work procedures
   d) Be the group moderator
7) The heights of the triangle you learned to construct:
   a) Seeing how the teacher constructs them
   b) Listening to the explanation of a parent or a friend at home
   c) By reading the instructions on how to build, in the student book.
   d) Trying several times yourself to build them
8) During project-based learning hours, in your group, you deal with:
   a) Drawing graphs, tables, illustrative figures
   b) Arguing all the work we have done as a group, you are the leader of the group

c) The collection of results and their processing, as well as the theoretical argumentation that is made based on the material in the text.
d) Providing the material basis (sometimes also the construction), of the part of the project that belongs to our group

9) Your preparation for math tests focuses mainly on:
a) Construction of graphics, figures, and sketches that enable me to repeat the exam
b) developing as many quizzes as can be found online
c) Repetition of as many types of exercises found in different books
d) The connection that the exercises and problems you will study with real life have.

10) Your portfolio works file has the maximum evaluation, in the tasks that:
a) Have to do with graphic illustrations of the knowledge gained
b) Relate to the arguments to be made for solving certain exercises
c) Have to do with extracting the main issues after the book has been studied
d) Have to do with figurative constructions of bodies as well as their combinations, for example, origami, tangram, etc.

Result:	More A	More B	More C	More D
	Visual	Auditor	Verbal	Kinesthetics

APPENDIX 4: STUDENT QUESTIONNAIRE ON THE USE OF ICT TOOLS IN PERSONALIZED LEARNING AND TEACHING IN MATHEMATICS.

You have developed during this period mathematics lessons assisted by ICT tools. Please, readthis questionnaire carefully and give your answers honestly.

Your opinion is very valuable to improve our work.

Thank you!

A. Personal data

1. Your age in years is:

- □ 11
- □ 12
- □ 13
- □ 14
- □ 15

2. Your gender

- □ Female
- □ Male

B. Access to ICTs tools

3. You have for personal use

- □ Computer only
- □ Only smartphones
- □ Only tablet
- □ Both tablets and smartphones
- □ Both computer and tablet
- □ Both smartphones and computers
- □ Both computers and smartphones and tablets
- □ Other (specify)

4. Do you have internet access?

- □ Yes
- □ No

5. How much time do you spend, on average per day with the technological equipment you own?

- □ None
- □ Maximum 1 hour
- □ From 1-2 hours
- □ From 2-3 hours
- □ From 3-4 hours
- □ More than 4 hours

C. Use of ICT tools owned by students to learn mathematics

6. How often do you use your personal computer, tablet, or smartphone to perform the following activities?

Activity You Do	Never	Rarely	Sometimes	Often	Always
To follow tutorials that explain mathematical concepts or exercises.					
To play mathematical games					
To copy math assignments sent to me via WhatsApp, or other communication networks.					
To install applications that help me to solve homework.					
To follow math tricks or to expand my knowledge of the world of math.					
To play fun games with a mathematical character.					
To make presentations of mathematical projects that we are obliged to do.					
To send homework by email to the math teacher or to upload them to the virtual platform we work with.					
To be discussed in mathematical forums.					
To follow Facebook or Instagram pages that have to do with math.					
To explain or listen to the solution of the exercises to my friends on the videocall.					

D. ICTs tools in math learning to increase students' self-esteem and motivation.

7.Show how the use of ICT tools during mathematics lessons influenced this period.

Using ICT tools to teach math	Not at all	Not really	Undecided	Somewhat	Very much
Increases the desire to learn mathematics					
Makes math an easier subject to understand					

Makes math more fun					
Strengthens cooperation between students					
Increases creativity and helps the imagination					
Increases students' self-confidence and self-esteem					
Illustrates the connection that mathematics has with the world around us					

8. How interesting was this math-based learning experience for you using ICT tools?

- □ Very bored
- □ Bored
- □ Uninterested
- □ Interested
- □ Very interested

9. Write down some reasons why you learn math.

10. Describe some components that should be included in a math lesson that you would like to go to.

APPENDIX 5: AGENDA OF THE TRAINING

20/03/2021	
Schedules	Program
08:30-11:00 11.00:11:45	Overview of math learning styles. Reflection 1 (Discussion) • Write down which learning styles you most often encounter with your students in math class. • Do you encounter it alone or in combination with others? • If combined with others, which styles are combined most often?
11.45 – 12.00	Coffee break
12:00 - 13:00	Discussion on reflection 1 (Discussion of the format of a test to identify learning styles)
13:00 – 14	Lunch break
14 -16:00	Technology and learning styles in mathematics. (Auditory, kinesthetic, visual, verbal style)
16:00-16:15	Coffee break
16: 15 -17:30	Reflection 2 (Discussion) Based on these learning styles, in groups of 5 people, choose a specific learning topic in math and plan a lesson where you adapt your technology-based teaching strategies to the three students' learning styles.
21/03/2021	
Ora	Program
08:30 -11:00 11.00-11:45	Personalized learning in mathematics Discussion Elements that a Personal Profile Learner (PPL) should have Discussion of a Personal Profile Learner (PPL) format
11.45 – 12.00	Coffee break
12:00 - 13:30	The role of ICT in personalized learning in mathematics

	Smart Board, its use.
13:30 – 14:30	Lunch break
14:30-15:30	Geo Gebra and its role in personalized teaching in mathematics. Various examples of how we can use GeoGebra in teaching mathematics.
15: 30 -15:45	Coffee break
15 : 45 - 16:45	The role of technological games in learning mathematics. Kahoot! application!
16:45 – 17: 30	Conclusion. Completion of the questionnaire on teachers' feedback regarding the training.

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