On the perturbations of the spectrum for upper-triangular operator
matrices

Abstract

A question asked by Du and Pan (in "Perturbation of Spectrums of 2×2 Operator Matrices", Proceedings of the American Mathematical Society, Vol. 121, 1994) about the perturbation of spectra for operator matrices of the form $M_C = \begin{bmatrix} A & C \\ 0 & B \end{bmatrix}$, where $A \in \mathcal{B}(\mathcal{X})$ and $B \in \mathcal{B}(\mathcal{Y})$ are given operators, and $C \in \mathcal{B}(\mathcal{Y}, \mathcal{X})$ where $\mathcal{X}$ and $\mathcal{Y}$ are Banach spaces will be given a positive answer, that is we prove that there exists an operator $C_0 \in \mathcal{B}(\mathcal{Y}, \mathcal{X})$ such that

$$\sigma(M_{C_0}) = \bigcap_{C \in \mathcal{B}(\mathcal{Y}, \mathcal{X})} \sigma(M_C).$$